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OF

WRITTEN ARITHMETIC

FOR

GRADED SCHOOLS.

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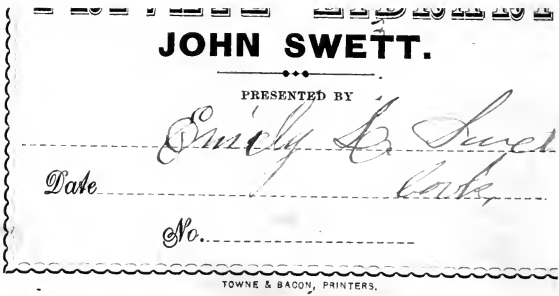

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ROBINSON'S MATHEMATICAL SERIES.

THE

RUDIMENTS

OF

WRITTEN ARITHMETIC:

CONTAINING

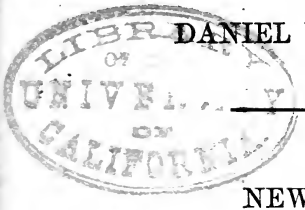
SLATE AND BLACK-BOARD EXERCISES FOR BEGINNERS,

AND DESIGNED FOR

GRADED SCHOOLS.

EDITED BY

DANIEL W. FISH, A.M.



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PREFACE.

In the preparation of this work, a special object has been kept in view by the author, namely; to furnish a small and simple class book for beginners, which shall contain no more of theory than is necessary for the illustration and application of the elementary principles of written arithmetic, applied to numerous, easy, and practical examples, and which shall be introductory to a full and complete treatise on this subject.

This book is not to be regarded as a *necessary* part of the Arithmetical Series by the same author, as the four books already composing that Series are believed to be properly and scientifically graded, and eminently adapted to *general* use; but this work has been prepared to meet a limited demand, in large *graded* schools, and in the public schools of New York, and similar cities, where a large number of pupils often obtain but a limited knowledge of arithmetic, and wish to commence its study quite young; and it is also designed for those who desire a larger number of simple and easy exercises for the slate and blackboard than are usually found in a complete work on written arithmetic, so that the beginner may acquire facility, promptness, and accuracy in the application and operations of the fundamental principles of this science.

The principles, definitions, rules, and applications so far as developed in this work coincide with the other books of the same series. Many of the Contractions, and special applications of the rules, particularly those that are at all difficult, have been omitted, and also the treatment of Denominate Fractions, and Decimals, all of which are fully and practically treated in the Progressive Practical, and the Higher Arithmetic. A few easy and practical applications of Cancellation, Analysis, Per centage and Simple Interest have been given, and a very large number of easy examples.

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UNIVERSITY OF CALIFORNIA

RUDIMENTS OF ARITHMETIC.

DEFINITIONS.

1. Quantity is any thing that can be increased, diminished, or measured; as distance, space, weight, motion, time.

2. A **Unit** is one, a single thing, or a definite quantity.

3. A **Number** is a unit, or a collection of units.

4. An **Abstract Number** is a number used without reference to any particular thing or quantity; as 3, 24, 756.

5. A **Concrete Number** is a number used with reference to some particular thing or quantity; as 21 hours, 4 cents, 230 miles.

6. A **Simple Number** is either an abstract number, or a concrete number of but one denomination; as 48, 52 pounds, 36 days.

7. A **Compound Number** is a concrete number expressed in two or more denominations; as 4 bushels 3 pecks, 8 rods 4 yards 2 feet 3 inches.

8. An **Integral Number**, or **Integer**, is a number which expresses whole things; as 5, 12 dollars, 17 men.

9. A **Fractional Number**, or **Fraction**, is a number which expresses equal parts of a whole thing or quantity; as $\frac{1}{2}$, $\frac{3}{4}$ of a pound, $\frac{5}{12}$ of a bushel.

10. Like Numbers have the same kind of unit, or express the same kind of quantity. Thus, 74 and 16 are like numbers; so are 74 pounds, 16 pounds, and 12 pounds; also, 4 weeks 3 days, and 16 minutes 20 seconds, both being used to express units of time.

11. **Unlike Numbers** have different kinds of units, or are used to express different kinds of quantity. Thus, 36 miles, and 15 days; 5 hours 36 minutes, and 7 bushels 3 pecks.

12. **Arithmetic** is the *Science* of numbers, and the *Art* of computation.

13. The **Five Fundamental Operations** of Arithmetic are, Notation and Numeration, Addition, Subtraction, Multiplication, and Division.

NOTATION AND NUMERATION.

14. **Notation** is a method of *writing* or expressing numbers by characters; and,

15. **Numeration** is a method of *reading* numbers expressed by characters.

16. Two systems of Notation are in general use—the *Roman* and *Arabic*.

THE ROMAN NOTATION

17. Employs seven capital letters to express numbers, thus:

Letters,	I	V	X	L	C	D	M
Values,	one,	five,	ten,	fifty,	one hundred,	five hundred,	one thousand.

18. The Roman notation is founded upon the following principles:

1st. Repeating a letter repeats its value. Thus, II represents two, XX twenty, CCC three hundred.

2d. If a letter of any value be placed *after* one of greater value, its value is to be *united to* that of the greater. Thus, XI represents eleven, LX sixty, DC six hundred.

3d. If a letter of any value be placed *before* one of greater

value, its value is to be *taken from* that of the greater. Thus, IX represents nine, XL forty, CD four hundred.

4th. If a letter of any value be placed *between* two letters, each of greater value, its value is to be *taken from* the *united value* of the other two. Thus, XIV represents fourteen, XXIX twenty-nine, XCIV ninety-four.

TABLE OF ROMAN NOTATION.

I	is One.	XVIII	is Eighteen.
II	" Two.	XIX	" Nineteen.
III	" Three.	XX	" Twenty.
IV	" Four.	XXI	" Twenty-one.
V	" Five.	XXX	" Thirty.
VI	" Six.	XL	" Forty.
VII	" Seven.	L	" Fifty.
VIII	" Eight.	LX	" Sixty.
IX	" Nine.	LXX	" Seventy.
X	" Ten.	LXXX	" Eighty.
XI	" Eleven.	XC	" Ninety.
XII	" Twelve.	C	" One hundred.
XIII	" Thirteen.	CC	" Two hundred.
XIV	" Fourteen.	D	" Five hundred.
XV	" Fifteen.	DC	" Six hundred.
XVI	" Sixteen.	M	" One thousand.
XVII	" Seventeen.		

Express the following numbers by the Roman notation:

1. Fourteen.
2. Nineteen.
3. Twenty-four.
4. Thirty-nine.
5. Forty-six.
6. Fifty-one.
7. Eighty-eight.
8. Seventy-three.
9. Ninety-five.
10. One hundred one.

THE ARABIC NOTATION

19. Employs ten characters or figures to express numbers.

Thus,

Figures,	0	1	2	3	4	5	6	7	8	9
Names and values,	naught, or cipher,	one,	two,	three,	four,	five,	six,	seven,	eight,	nine.

20. The first character is called *naught*, because it has no value of its own. The other nine characters are called *significant figures*, because each has a value of its own.

21. As we have no single character to represent ten, we express it by writing the unit, 1, at the left of the cipher, 0, thus, 10. In the same manner we represent

2 tens, or twenty,	3 tens, or thirty,	4 tens, or forty,	5 tens or fifty,	6 tens, or sixty,	7 tens, or seventy,	8 tens, or eighty,	9 tens, or ninety,
20;	30;	40;	50;	60;	70;	80;	90.

22. When a number is expressed by two figures, the right hand figure is called *units*, and the left hand figure *tens*.

We express the numbers between 10 and 20, thus:

eleven,	twelve,	thirteen,	fourteen,	fifteen,	sixteen,	seventeen,	eighteen,	nineteen.
11,	12,	13,	14,	15,	16,	17,	18,	19.

In like manner we express the numbers between 20 and 30, thus: 21, 22, 23, 24, 25, 26, 27, 28, 29, &c.

The greatest number that can be expressed by *two* figures is 99.

23. We express one hundred by writing the unit, 1, at the left hand of two ciphers; thus, 100. In like manner we write two hundred, three hundred, &c., to nine hundred. Thus:

one hundred,	two hundred,	three hundred,	four hundred,	five hundred,	six hundred,	seven hundred,	eight hundred,	nine hundred.
100,	200,	300,	400,	500,	600,	700,	800,	900.

24. When a number is expressed by three figures, the right hand figure is called *units*, the second figure *tens*, and the left hand figure *hundreds*.

As the ciphers have, of themselves, no value, but are always used to denote the absence of value in the places they occupy, we express tens and units with hundreds, by writing, in place of the ciphers, the numbers representing the tens and units. To express one hundred fifty, we write 1 hundred, 5 tens, and 0 units; thus, 150. To express seven hundred ninety-two, we write 7 hundreds, 9 tens, and 2 units; thus,

hundreds.	tens.	units.
7	9	2

The greatest number that can be expressed by *three* figures is 999.

Express the following numbers by figures:—

1. Write one hundred twenty-five.
2. Write four hundred eighty-three.
3. Write seven hundred sixteen.
4. Express by figures nine hundred.
5. Express by figures two hundred ninety.
6. Write eight hundred nine.
7. Write five hundred five.
8. Write five hundred fifty-seven.

25. We express one thousand by writing the unit, 1, at the left hand of three ciphers; thus, 1000. In the same manner we write two thousand, three thousand, &c., to nine thousand; thus,

one thousand,	two thousand,	three thousand,	four thousand,	five thousand,	six thousand,	seven thousand,	eight thousand,	nine thousand.
1000,	2000,	3000,	4000,	5000,	6000,	7000,	8000,	9000.

26. When a number is expressed by four figures, the places, commencing at the right hand, are *units, tens, hundreds, thousands.*

To express hundreds, tens, and units with thousands, we write in each place the figure indicating the number we wish to express in that place. To write four thousand two hundred sixty-nine, we write 4 in the place of thousands, 2 in the place of hundreds, 6 in the place of tens, 9 in the place of units; thus,

thousands.	hundreds.	tens.	units.
4	2	6	9

The greatest number that can be expressed by *four* figures is 9999.

Express the following numbers by figures:—

1. One thousand two hundred.
2. Five thousand one hundred sixty.
3. Three thousand seven hundred forty-one.
4. Eight thousand fifty six.
5. Two thousand ninety.
6. Seven thousand nine.
7. One thousand one.
8. Nine thousand four hundred twenty-seven.
9. Four thousand thirty-five.
10. One thousand nine hundred four.

Read the following numbers:—

11. 76; 128; 405; 910; 116; 3414; 1025.
12. 2100; 5047; 7009; 4670; 3997; 1001.

27. Next to thousands come *tens* of thousands, and next to these come *hundreds* of thousands, as tens and hundreds come in their order after units.

Ten thousand is expressed by removing the unit, 1, one place to the left of the place of thousands, or by writing it

at the left hand of four ciphers; thus, 10000; and one hundred thousand is expressed by removing the unit, 1, still one place further to the left, or by writing it at the left hand of five ciphers; thus, 100000. We can express thousands, tens of thousands, and hundreds of thousands in one number, in the same manner as we express units, tens, and hundreds in one number. To express five hundred twenty-one thousand eight hundred three, we write five in the sixth place, counting from units, 2 in the fifth place, 1 in the fourth place, 8 in the third place, 0 in the second place, (because there are no tens), and 3 in the place of units; thus,

hundreds of thousands.	tens of thousands.	thousands.	hundreds.	tens.	units.
5	2	1	8	0	3

The greatest number that can be expressed by *five* figures is 99999; and by *six* figures, 999999.

Write the following numbers in figures:—

1. Twenty thousand.
2. Forty-seven thousand.
3. Eighteen thousand one hundred.
4. Twelve thousand three hundred fifty.
5. Thirty-nine thousand five hundred twenty-two.
6. Fifteen thousand two hundred six.
7. Eleven thousand twenty-four.
8. Forty thousand ten.
9. Sixty thousand six hundred.
10. Two hundred twenty thousand.
11. One hundred fifty-six thousand.
12. Eight hundred forty thousand three hundred.

Read the following numbers:

13. 5006; 12304; 96071; 5470; 203410.

14. 36741; 400560; 13061; 49000; 100010.

For convenience in reading large numbers, we may point them off, by commas, into periods of three figures each, counting from the right hand or unit figure. This pointing enables us to read the hundreds, tens, and units in each period with facility as seen in the following

NUMERATION TABLE.

hundreds of trillions	tens	units	hundreds of billions	tens	units	hundreds of millions	tens	units	hundreds of thousands	tens	units	hundreds of units	tens	units
8	7	6,	5	5	6,	7	8	9,	0	1	2,	3	4	5
fifth period.			fourth period.			third period.			second period.			first period.		

28. Figures occupying different places in a number, as units, tens, hundreds, &c., are said to express different orders of units.

29. In *numerating*, or expressing numbers verbally, the various orders of units have the following names:

ORDERS.	NAMES.
1st order is called	Units.
2d order " "	Tens.
3d order " "	Hundreds.
4th order " "	Thousands.
5th order " "	Tens of thousands.
6th order " "	Hundreds of thousands.
7th order " "	Millions.
8th order " "	Tens of millions.
9th order " "	Hundreds of millions.
&c., &c.	&c., &c.

Write and read the following numbers:—

1. One unit of the third order, two of the second, five of the first. *Ans.* 125; read, *one hundred twenty-five.*

2. Two units of the 5th order, four of the 4th, five of the 2d, six of the 1st.

Ans. 24056; read, *twenty-four thousand fifty-six.*

3. Seven units of the 4th order, five of the third, three of the 2d, eight of the 1st.

4. Two units of the 7th order, nine of the 6th, four of the 3d, one of the 1st, seven of the 2d.

5. Three units of the 6th order, four of the 2d.

6. Nine units of the 8th order, six of the 7th, three of the 5th, seven of the 4th, nine of the 1st.

7. Four units of the 10th order, six of the 8th, four of the 7th, two of the 6th, one of the 3d, five of the 2d.

8. Eight units of the 12th order, four of the 11th, six of the 10th, nine of the 7th, three of the 6th, five of the 5th, two of the 3d, eight of the 1st.

30. Since the number expressed by any figure depends upon the place it occupies, it follows that figures have two values, Simple and Local.

31. The **Simple Value** of a figure is its value when taken alone; thus, 4, 7, 2.

32. The **Local Value** of a figure is its value when used with another figure or figures in the same number. Thus, in 325, the local value of the 3 is 300, of the 2 is 20, and of the 5 is 5 units.

NOTE.—When a figure occupies units' place, its simple and local values are the same.

33. The leading principles upon which the Arabic notation is founded are embraced in the following

GENERAL LAWS.

I. *All numbers are expressed by applying the ten figures to the different orders of units.*

II. *The different orders of units increase from right to left, and decrease from left to right, in a tenfold ratio.*

III. *Every removal of a figure one place to the left, increases its local value tenfold; and every removal of a figure one place to the right, diminishes its local value tenfold.*

From this analysis of the principles of Notation and Numeration, we derive the following rules:—

RULE FOR NOTATION.

I. *Beginning at the left hand, write the figures belonging to the highest period.*

II. *Write the hundreds, tens, and units, of each successive period in their order, placing a cipher wherever an order of units is omitted.*

RULE FOR NUMERATION.

I. *Separate the number into periods of three figures each, commencing at the right hand.*

II. *Beginning at the left hand, read each period separately, and give the name to each period, except the last, or period of units.*

34. Until the pupil can write numbers readily, it may be well for him to write several periods of ciphers, point them off, over each period write its name, thus,

Trillions,	Billions,	Millions,	Thousands,	Units.
000,	000,	000,	000,	000

and then write the given numbers underneath, in their appropriate places.

EXERCISES IN NOTATION AND NUMERATION.

Express the following numbers by figures: —

1. Four hundred thirty-six.
2. Seven thousand one hundred sixty-four.
3. Twenty-six thousand twenty-six.
4. Fourteen thousand two hundred eighty.
5. One hundred seventy-six thousand.
6. Four hundred fifty thousand thirty-nine.
7. Ninety-five million.
8. Four hundred eighty-three million eight hundred sixteen thousand one hundred forty-nine.
9. Nine hundred thousand ninety.
10. Ten million ten thousand ten hundred ten.

Point off, numerate, and read the following numbers: —

11. 8240.	15. 111111.	19. 370005.
12. 400900.	16. 57468139.	20. 9400706342.
13. 308.	17. 5628.	21. 38429526.
14. 60720.	18. 11111111.	22. 11111111111.

23. Write seven million thirty-six.
24. Write five hundred sixty-three thousand four.
25. Write one million ninety-six thousand.
26. A certain number contains 3 units of the seventh order, 6 of the fifth, 4 of the fourth, 1 of the third, 5 of the second, and 2 of the first; what is the number?
27. What orders of units are contained in the number 290648?

ADDITION.

35. **Addition** is the process of uniting *several* numbers of the same kind into *one* equivalent number.

36. The **Sum**, or **Amount**, is the result obtained.

ADDITION TABLE.

2 and 1 are 3	3 and 1 are 4	4 and 1 are 5	5 and 1 are 6
2 and 2 are 4	3 and 2 are 5	4 and 2 are 6	5 and 2 are 7
2 and 3 are 5	3 and 3 are 6	4 and 3 are 7	5 and 3 are 8
2 and 4 are 6	3 and 4 are 7	4 and 4 are 8	5 and 4 are 9
2 and 5 are 7	3 and 5 are 8	4 and 5 are 9	5 and 5 are 10
2 and 6 are 8	3 and 6 are 9	4 and 6 are 10	5 and 6 are 11
2 and 7 are 9	3 and 7 are 10	4 and 7 are 11	5 and 7 are 12
2 and 8 are 10	3 and 8 are 11	4 and 8 are 12	5 and 8 are 13
2 and 9 are 11	3 and 9 are 12	4 and 9 are 13	5 and 9 are 14
2 and 10 are 12	3 and 10 are 13	4 and 10 are 14	5 and 10 are 15
2 and 11 are 13	3 and 11 are 14	4 and 11 are 15	5 and 11 are 16
2 and 12 are 14	3 and 12 are 15	4 and 12 are 16	5 and 12 are 17
6 and 1 are 7	7 and 1 are 8	8 and 1 are 9	9 and 1 are 10
6 and 2 are 8	7 and 2 are 9	8 and 2 are 10	9 and 2 are 11
6 and 3 are 9	7 and 3 are 10	8 and 3 are 11	9 and 3 are 12
6 and 4 are 10	7 and 4 are 11	8 and 4 are 12	9 and 4 are 13
6 and 5 are 11	7 and 5 are 12	8 and 5 are 13	9 and 5 are 14
6 and 6 are 12	7 and 6 are 13	8 and 6 are 14	9 and 6 are 15
6 and 7 are 13	7 and 7 are 14	8 and 7 are 15	9 and 7 are 16
6 and 8 are 14	7 and 8 are 15	8 and 8 are 16	9 and 8 are 17
6 and 9 are 15	7 and 9 are 16	8 and 9 are 17	9 and 9 are 18
6 and 10 are 16	7 and 10 are 17	8 and 10 are 18	9 and 10 are 19
6 and 11 are 17	7 and 11 are 18	8 and 11 are 19	9 and 11 are 20
6 and 12 are 18	7 and 12 are 19	8 and 12 are 20	9 and 12 are 21
10 and 1 are 11	11 and 1 are 12	12 and 1 are 13	13 and 1 are 14
10 and 2 are 12	11 and 2 are 13	12 and 2 are 14	13 and 2 are 15
10 and 3 are 13	11 and 3 are 14	12 and 3 are 15	13 and 3 are 16
10 and 4 are 14	11 and 4 are 15	12 and 4 are 6	13 and 4 are 17
10 and 5 are 15	11 and 5 are 16	12 and 5 are 17	13 and 5 are 18
10 and 6 are 16	11 and 6 are 17	12 and 6 are 18	13 and 6 are 19
10 and 7 are 17	11 and 7 are 18	12 and 7 are 19	13 and 7 are 20
10 and 8 are 18	11 and 8 are 19	12 and 8 are 20	13 and 8 are 21
10 and 9 are 19	11 and 9 are 20	12 and 9 are 21	13 and 9 are 22
10 and 10 are 20	11 and 10 are 21	12 and 10 are 22	13 and 10 are 23
10 and 11 are 21	11 and 11 are 22	12 and 11 are 23	13 and 11 are 24
10 and 12 are 22	11 and 12 are 23	12 and 12 are 24	13 and 12 are 25

MENTAL EXERCISES.

1. A farmer paid 6 dollars for a straw-cutter, and 9 dollars for a plow; how much did he pay for both?

ANALYSIS. He paid the sum of 6 dollars and 9 dollars, which is 15 dollars. Therefore, he paid 15 dollars for both.

2. John gave 4 apples to James, 8 to Henry, and 9 to Asa; how many did he give to all?

3. Gave 7 dollars for a barrel of flour, 9 dollars for a hundred weight of sugar, and 6 dollars for a tub of butter; how much did I give for the whole?

4. I have two pear trees; one tree produced 12 bushels of pears, and the other 11 bushels; how many bushels did both produce?

5. A man bought 4 cords of wood for 12 dollars, and 7 bushels of corn for 5 dollars; how much did he pay for both?

6. James gave 11 cents for a slate, and had 8 cents left; how many cents had he at first?

7. A lady paid 5 dollars for a bonnet, 10 dollars for a shawl, and had 7 dollars left; how much money had she at first?

8. In a shop are 8 men, 9 boys, and 6 girls, at work; how many persons are at work in the shop?

9. Rollin bought a quire of paper for 12 cents, a slate for 13 cents, and gave 10 cents to a beggar; how much money did he pay out in all?

10. A man bought 4 bushels of wheat for 7 dollars, 18 bushels of corn for 11 dollars, and 2 cords of wood for 5 dollars; how much did he pay for the whole?

11. A farmer has 6 cows in one yard, 9 in another, and as many in the third yard as in both the others; how many cows has he?

PROMISCUOUS ADDITION TABLE.

2 and 5 are how many?	7 and 9 are how many?
6 and 2 are how many?	6 and 5 are how many?
2 and 4 are how many?	3 and 6 are how many?
8 and 9 are how many?	4 and 4 are how many?
9 and 4 are how many?	7 and 8 are how many?
4 and 7 are how many?	9 and 3 are how many?
8 and 6 are how many?	5 and 4 are how many?
6 and 3 are how many?	3 and 8 are how many?
7 and 2 are how many?	5 and 6 are how many?

3 and 9 are how many?	5 and 8 are how many?
4 and 5 are how many?	3 and 7 are how many?
9 and 8 are how many?	6 and 4 are how many?
8 and 5 are how many?	7 and 6 are how many?
4 and 9 are how many?	6 and 8 are how many?
5 and 4 are how many?	9 and 5 are how many?
2 and 7 are how many?	8 and 3 are how many?
7 and 5 are how many?	9 and 6 are how many?
5 and 2 are how many?	5 and 7 are how many?

6 and 9 are how many?	4 and 6 are how many?
7 and 7 are how many?	7 and 3 are how many?
3 and 4 are how many?	2 and 8 are how many?
8 and 7 are how many?	5 and 9 are how many?
4 and 8 are how many?	8 and 8 are how many?
9 and 2 are how many?	6 and 7 are how many?
5 and 3 are how many?	5 and 5 are how many?
6 and 6 are how many?	9 and 7 are how many?
7 and 4 are how many?	9 and 9 are how many?

37. The **Sign of Addition** is the perpendicular cross, +, called *plus*. It indicates that the numbers connected by it are to be added; as $3 + 5 + 7$, read 3 plus 5 plus 7.

38. The **Sign of Equality** is two short, parallel, horizontal lines, =. It indicates that the numbers, or combination of numbers, connected by it are equal; as $4 + 8 = 9 + 3$, read the sum of 4 plus 8 is equal to the sum of 9 plus 3

CASE I.

39. When the amount of each column is less than 10.

1. A drover bought three flocks of sheep. The first contained 232, the second 422, and the third 245; how many did he buy in all?

OPERATION.

hunds.	tens.	units.
2	3	2
4	2	2
2	4	5
Amount, 8	9	9

ANALYSIS. We arrange the numbers so that units of like order shall stand in the same column. We then add the columns separately, for convenience commencing at the right hand, and write each result under the column added. Thus, we have 5 and 2 and 2 are 9, the sum of the units; 4 and 2 and 3 are 9, the sum of the tens; 2 and 4 and 2 are 8, the sum of the hundreds.—Hence, the entire amount is 8 hundreds 9 tens and 9 units, or 899, the Answer.

EXAMPLES FOR PRACTICE.

(2.)	(3.)	(4.)	(5.)
403	164	510	234
271	321	176	324
124	510	203	140
Ans. 798			

(6.)	(7.)	(8.)	(9.)
1234	2041	3102	4100
2405	3216	2253	1523
5140	1500	4014	2041
Ans. 8779			

10. What is the sum of 421, 305 and 5162?

11. What is the sum of 3121, 436 and 2002?

CASE II.

40. When the amount of any column equals or exceeds 10.

1. A merchant pays 397 dollars for freights, 476 dollars for a clerk, and 873 for rent of a store; what is the amount of his expenses?

OPERATION.

397
476
873
1746

ANALYSIS. We arrange the numbers so that units of like order shall stand in the same column. We then add the first, or right hand column, and find the sum to be 16 units, or 1 ten and 6 units; writing the 6 units under the column of units, we add the 1 ten to the column of tens, and find the sum to be 24 tens, or 2 hundreds and 4 tens; writing the 4 tens under the column of tens, we add the 2 hundreds to the column of hundreds, and find the sum to be 17 hundreds, or 1 thousand and 7 hundreds; writing the 7 hundreds under the column of hundreds, and the 1 in thousands' place, we have the entire sum, 1746.

NOTES.—1. In adding, learn to pronounce the partial results without naming the *figures* separately. Thus, in the operation given for illustration, say 3, 9, 16; 8, 15, 24; 10, 14, 17.

2.—When the sum of any column is greater than 9, the process of adding the tens to the next column is called *carrying*.

41. From the preceeding examples and illustrations we deduce the following

RULE. I. *Write the numbers to be added so that all the units of the same order shall stand in the same column; that is, units under units, tens under tens, &c.*

II. *Commencing at units, add each column separately, and write the sum underneath, if it be less than ten.*

III. *If the sum of any column be ten or more than ten, write the unit figure only, and add the ten or tens to the next column.*

IV. *Write the entire sum of the last column.*

PROOF. Begin with the right hand or unit column, and add the figures in each column in an opposite direction from that in which they were first added; if the two results agree, the work is supposed to be right.

EXAMPLES FOR PRACTICE.

(1.) inches.	(2.) feet.	(3.) pounds.	(4.) yards.	(5.) miles.
142	325	75	407	1270
325	46	276	96	342
476	674	508	2584	79
943	1045	859	3087	1691

(6.)	(7.)	(8.)	(9.)	(10.)
842	376	426	713	4761
396	407	397	86	374
472	862	450	345	83
205	94	294	60	19

11. What is the sum of $912 + 342 + 187 + 46$?

Ans. 1487.

12. What is the sum of $214 + 425 + 90 + 37$?

Ans. 766.

13. What is the sum of 56 feet, 450 feet, and 680 feet?

Ans. 1186 feet.

14. What is the sum of 1942 dollars, and 685 dollars?

15. A man paid 375 dollars for a span of horses, 160 dollars for a carriage, and 87 dollars for a harness; how much did he pay for all? *Ans.* 622 dollars.

16. A man traveled 476 miles by railroad, 390 miles by steamboat, and 120 miles by stage; how many miles in all, did he travel? *Ans.* 986.

17. A carpenter built a house for 2464 dollars, a barn for 496 dollars, and outhouses for 309 dollars; how much did he receive for building all?

18. A merchant bought at public auction 520 yards of broadcloth, 386 yards of muslin, 92 yards of flannel, and 156 yards of silk; how many yards in all?

19. A father divided his estate among his four sons, giving each 2087 dollars; what was the amount of his estate?

20. Three persons deposited money in a bank; the first 4780 dollars, the second 3042 dollars, and the third 407 dollars; how much did they all deposit?

21. Five men engage in business as partners, and each puts in 2375 dollars; what is the whole amount of capital invested? *Ans.* 11875 dollars.

(22.)	(23.)	(24.)	(25.)
765	347	630	4603
381	192	815	7106
976	763	456	972
315	410	307	385
169	507	960	64

Ans. 2606

(26.)	(27.)	(28.)
767346	374205	4076315
432761	108497	5632870
386109	643024	8219634
508763	879638	3827692

Ans. 2094979

29. $3720 + 647 + 190 + 82 =$ how many?

Ans. 4639.

30. $962 + 2161 + 500 + 75 =$ how many?

Ans. 3698.

31. $4170 + 1009 + 642 + 120 + 18 =$ how many?

32. $3000 + 47602 + 805 + 1266 + 76 =$ how many?

33. $69 + 4030 + 349 + 1384 + 72 + 400 =$ how many?

34. What is the sum of two thousand eight hundred fifty-six, twelve thousand eighty-four, seven hundred forty-two, and sixty-nine? *Ans.* 14751.

35. What is the amount of twenty thousand five hundred ten, six thousand nine hundred forty-four, and three thousand two hundred? *Ans.* 30654.

36. What is the sum of forty-seven thousand fifty, nine thousand one hundred six, fourteen hundred ninety-two, and five hundred twelve? *Ans.* 58160.

37. What is the sum of one hundred forty thousand three hundred thirty-four, seventy nine-thousand six hundred five, twenty-five hundred twenty-five, and three thousand sixty-nine? *Ans.* 225533.

38. What is the amount of five hundred thousand five hundred five, eighty-four thousand two hundred, fifteen thousand six hundred twenty, and seventeen hundred seventeen? *Ans.* 602042.

39. How many men in an army consisting of 26840 infantry, 6370 cavalry, 3250 dragoons, 750 artillery, and 320 miners? *Ans.* 37530.

40. A merchant deposited 125 dollars in bank on Monday, 91 on Tuesday, 164 on Wednesday, 200 on Thursday, 196 on Friday, and 73 on Saturday; how much did he deposit during the week?

41. By selling a farm for 3586 dollars, 684 dollars were lost; how much did the farm cost?

42. If I were born in 1840, when will I be 63 years old?

43. A man willed his estate to his wife, two sons and three daughters; to his daughters he gave 1565 dollars apiece, to his sons 3560 dollars each, and to his wife 4720 dollars; how much was his estate? *Ans.* 16535 dollars.

44. A man engaging in trade, gained 450 dollars the first

year, 684 dollars the second, and as much the third as he gained during the first and second; how much was his whole gain? *Ans.* 2268 dollars.

45. I bought three village lots for 12570 dollars, and sold them so as to gain 745 dollars on each lot; for how much did I sell them? *Ans.* 14805 dollars.

46. A has 3240 dollars, B has 5672 dollars, and C has 1000 more than A and B together; how many dollars have all? *Ans.* 18824 dollars.

47. A man was 32 years old when his son was born; how old will he be when his son is 36 years old? *Ans.* 68 years.

48. The Old Testament contains 39 books, 929 chapters, 23214 verses, 592439 words, and 2728100 letters; the New Testament contains 37 books, 269 chapters, 7959 verses, 181153 words, and 838380 letters; what is the total number of each in the Bible?

Ans. 76 books, 1198 chapters, 31173 verses, 773592 words, and 3566480 letters.

49. The number of immigrants landed in New York in 1858 was 78589, in 1859, 79322, and in 1860, 103621; what was the total number landed in the three years?

Ans. 261532.

50. In 1860, the population of New York was 814277, of Philadelphia 568034, of Boston 177902, of New Orleans 170766, of St. Louis 162179, of Chicago 109429, and of Cincinnati 160000; what was the total population of these cities? *Ans.* 2162587

51. In the year 1856, the United States exported molasses to the value of 154630 dollars; in 1857, 108003 dollars; in 1858, 115893 dollars; what was the value of the molasses exported in those three years? *Ans.* 378526 dollars.

52. During the same years, respectively, the United States

exported tobacco to the value of 1829207 dollars, 1458553 dollars, and 2410224 dollars; what was the total value of the tobacco exported in those years? *Ans.* 5697984 dollars.

53. How many miles from the southern extremity of Lake Michigan to the Gulf of St. Lawrence, passing through Lake Michigan, 330 miles; Lake Huron, 260 miles; River St. Clair, 24 miles; Lake St. Clair, 20 miles; Detroit River, 23 miles; Lake Erie, 260 miles; Niagara River, 34 miles; Lake Ontario, 180 miles; and the River St. Lawrence, 750 miles? *Ans.* 1881 miles.

54. At the commencement of the year 1858 there were in operation in the New England States, 3751 miles of railroad; in New York, 2590 miles; in Pennsylvania, 2546; in Ohio, 2946; in Virginia, 1233; in Illinois, 2678; and in Georgia, 1233; what was the aggregate number of miles in operation in all these States? *Ans.* 16977.

55. The number of pieces of silver coin made at the United States Mint at Philadelphia in the year 1858, were as follows: 4628000 half dollars, 10600000 quarter dollars, 690000 dimes, 4000000 half dimes, and 1266000 three-cent pieces; what was the total number of pieces coined? *Ans.* 21184000.

(56.)	(57.)	(58.)	(59.)
344	843	1186	81988
388	738	513	380167
613	237	740	108424
803	218	1820	193686
825	347	955	144225
412	288	736	112558
322	483	810	107481
886	753	511	176826
620	834	1179	145851
5213		8450	1451206

(60.)	(61.)	(62.)	(63.)
35938	47197	12380	456568
49172	63956	98795	754712
56546	85678	23442	567346
82564	35495	87639	543678
69789	16457	91758	342766
47321	94667	19347	768345
77563	76463	81731	563875
83563	34698	29342	547427
54973	17179	75659	945956
38137	93965	35446	165675
54246	81367	98237	756431
95864	29787	12845	354747
48135	79826	87677	543864
37975	31275	23444	567456
48467	59689	39878	621367

(64.)	(65.)	(66.)	(67.)
768856	576654	987654	9873785
674387	678456	123456	1239564
978874	754543	876864	7591074
567678	786567	234246	3517569
568594	964432	765183	8598674
639678	699678	345927	2513756
669657	978321	654678	3454210
594886	678789	456432	7656754
695756	564673	345719	5467856
789568	895437	765391	5645781
689689	569128	673123	7893344
638786	678982	437987	3216677
675968	869771	566789	4569911
958789	668339	544321	6543344
769896	956234	891389	9576677
153674	195876	219721	1539900
331767	957412	625247	6662233
355989	573375	431321	4235566
11522492	13046667	9945448	99796675

SUBTRACTION.

42. **Subtraction** is the process of determining the difference, between two numbers of the same unit value.

43. The **Difference** or **Remainder** is the result obtained.

SUBTRACTION TABLE.

1 from 2 leaves 1	2 from 3 leaves 1	3 from 4 leaves 1	4 from 5 leaves 1
1 from 3 leaves 2	2 from 4 leaves 2	3 from 5 leaves 2	4 from 6 leaves 2
1 from 4 leaves 3	2 from 5 leaves 3	3 from 6 leaves 3	4 from 7 leaves 3
1 from 5 leaves 4	2 from 6 leaves 4	3 from 7 leaves 4	4 from 8 leaves 4
1 from 6 leaves 5	2 from 7 leaves 5	3 from 8 leaves 5	4 from 9 leaves 5
1 from 7 leaves 6	2 from 8 leaves 6	3 from 9 leaves 6	4 from 10 leaves 6
1 from 8 leaves 7	2 from 9 leaves 7	3 from 10 leaves 7	4 from 11 leaves 7
1 from 9 leaves 8	2 from 10 leaves 8	3 from 11 leaves 8	4 from 12 leaves 8
1 from 10 leaves 9	2 from 11 leaves 9	3 from 12 leaves 9	4 from 13 leaves 9
1 from 11 leaves 10	2 from 12 leaves 10	3 from 13 leaves 10	4 from 14 leaves 10
5 from 6 leaves 1	6 from 7 leaves 1	7 from 8 leaves 1	8 from 9 leaves 1
5 from 7 leaves 2	6 from 8 leaves 2	7 from 9 leaves 2	8 from 10 leaves 2
5 from 8 leaves 3	6 from 9 leaves 3	7 from 10 leaves 3	8 from 11 leaves 3
5 from 9 leaves 4	6 from 10 leaves 4	7 from 11 leaves 4	8 from 12 levves 4
5 from 10 leaves 5	6 from 11 leaves 5	7 from 12 leaves 5	8 from 13 leaves 5
5 from 11 leaves 6	6 from 12 leaves 6	7 from 13 leaves 6	8 from 14 leaves 6
5 from 12 leaves 7	6 from 13 leaves 7	7 from 14 leaves 7	8 from 15 leaves 7
5 from 13 leaves 8	6 from 14 leaves 8	7 from 15 leaves 8	8 from 16 leaves 8
5 from 14 leaves 9	6 from 15 leaves 9	7 from 16 leaves 9	8 from 17 leaves 9
5 from 15 leaves 10	6 from 16 leaves 10	7 from 17 leaves 10	8 from 18 leaves 10
9 from 10 leaves 1	10 from 11 leaves 1	11 from 12 leaves 1	12 from 13 leaves 1
9 from 11 leaves 2	10 from 12 leaves 2	11 from 13 leaves 2	12 from 14 leaves 2
9 from 12 leaves 3	10 from 13 leaves 3	11 from 14 leaves 3	12 from 15 leaves 3
9 from 13 leaves 4	10 from 14 leaves 4	11 from 15 leaves 4	12 from 16 leaves 4
9 from 14 leaves 5	10 from 15 leaves 5	11 from 16 leaves 5	12 from 17 leaves 5
9 from 15 leaves 6	10 from 16 leaves 6	11 from 17 leaves 6	12 from 18 leaves 6
9 from 16 leaves 7	10 from 17 leaves 7	11 from 18 leaves 7	12 from 19 leaves 7
9 from 17 leaves 8	10 from 18 leaves 8	11 from 19 leaves 8	12 from 20 leaves 8
9 from 18 leaves 9	10 from 19 leaves 9	11 from 20 leaves 9	12 from 21 leaves 9
9 from 19 leaves 10	10 from 20 leaves 10	11 from 21 leaves 10	12 from 22 leaves 10

MENTAL EXERCISES.

1. A grocer having 20 boxes of lemons, sold 12 boxes; how many boxes had he left?

ANALYSIS. He had left the difference between 20 boxes and 12 boxes, which is 8 boxes. Therefore, he had 8 boxes left.

2. If a man earn 12 dollars a week, and spend 7 for provisions, how many dollars has he left?

3. If I borrow 15 dollars, and pay 9 dollars, how many dollars remain unpaid?

4. John had 11 marbles, and lost 5 of them; how many had he left?

5. From a cistern containing 22 barrels of water, 9 barrels leaked out; how many barrels remained?

6. In a school are 24 boys and 12 girls; how many more boys than girls?

7. From a piece of cloth containing 17 yards, 8 yards were cut; how many yards remained?

8. Orin paid 15 dollars for a coat, and 9 dollars for a pair of pantaloons; how much more did he pay for the coat than for the pantaloons?

9. Cora is 23 years old, and her brother is 10 years younger; how old is her brother?

10. A jeweler bought a watch for 11 dollars, and sold it for 18 dollars; how much did he gain?

11. A boy gave 21 cents for some pictures, which were worth no more than 17 cents; how much more than their value did he give for them?

12. A grocer bought a barrel of sugar for 16 dollars, but it not proving as good as he expected, he sold it for 11 dollars; how much did he lose on it?

PROMISCUOUS SUBTRACTION TABLE.

5 from 14 how many?	6 from 14 how many?
5 from 9 how many?	8 from 15 how many?
9 from 10 how many?	5 from 11 how many?
6 from 7 how many?	7 from 10 how many?
7 from 12 how many?	3 from 13 how many?
9 from 12 how many?	9 from 11 how many?
5 from 10 how many?	6 from 12 how many?
6 from 11 how many?	8 from 10 how many?

8 from 9 how many?	4 from 11 how many?
7 from 16 how many?	3 from 10 how many?
2 from 11 how many?	5 from 12 how many?
5 from 8 how many?	7 from 13 how many?
9 from 14 how many?	8 from 12 how many?
9 from 13 how many?	9 from 16 how many?
7 from 9 how many?	6 from 13 how many?
2 from 10 how many?	4 from 12 how many?

7 from 15 how many?	8 from 16 how many?
8 from 17 how many?	9 from 15 how many?
4 from 10 how many?	7 from 11 how many?
7 from 14 how many?	3 from 12 how many?
3 from 11 how many?	6 from 15 how many?
5 from 13 how many?	9 from 18 how many?
9 from 17 how many?	6 from 10 how many?
8 from 14 how many?	4 from 13 how many?

44. The **Minuend** is the number to be subtracted from.

45. The **Subtrahend** is the number to be subtracted.

46. The **Sign of Subtraction** is a short horizontal line, —, called *minus*. When placed between two numbers, it denotes that the one after it is to be taken from the one before it. Thus, 8—6=2, is read 8 minus 6 equals 2, and denotes that 6, the *subtrahend*, taken from 8, the *minuend*, equals 2, the *remainder*.

CASE I.

47. When no figure in the subtrahend is greater than the corresponding figure in the minuend.

1. From 574 take 323.

OPERATION.	
Minuend,	574
Subtrahend,	323
Remainder,	251

ANALYSIS. We write the less number under the greater, with units under units, tens under tens, &c., and draw a line underneath. Then, beginning at the right hand, we subtract separately each figure of the subtrahend from the figure above it in the minuend. Thus, 3 from 4 leaves 1, which is the difference of the units; 2 from 7 leaves 5, the difference of the tens; 3 from 5 leaves 2, the difference of the hundreds. Hence, we have for the whole difference, 2 hundreds 5 tens and 1 unit, or 251.

EXAMPLES FOR PRACTICE.

	(1.)	(2.)	(3.)	(4.)
Minuend,	876	349	637	508
Subtrahend,	435	212	431	104
Remainder,	441	137	206	404

	(5.)	(6.)	(7.)	(8.)
	987	753	438	695
	647	502	421	535
	340	251	17	160

	(9.)	(10.)	(11.)	(12.)
From	7642	8730	2369	9786
Take	3211	6430	2104	3126
	4431	2300	265	6660

	Remainders.
13. From 4376 take 1254.	3122.
14. From 70342 take 50130.	20212.
15. From 137647 take 16215.	121432.
16. Subtract 32014 from 86325.	54311.
17. Subtract 217356 from 719568.	502212.
18. 437615 — 213502 = how many?	224113.
19. 732740 — 11520 = how many?	721220
20. 2042674 — 32142 = how many?	
21. 8461203 — 7161003 = how many?	

22. From three thousand two hundred seventy-six, take two thousand one hundred forty-three.

23. From one hundred eighty-three thousand four hundred sixty, take fifty-two thousand one hundred fifty.

Ans. 131310.

24. A man bought a piece of property for 7634 dollars, and sold the same for 3132 dollars; how much did he lose?

Ans. 4502 dollars.

25. A merchant sold goods to the amount of 41763 dollars, and by so doing gained 11521 dollars; how much did the goods cost him? *Ans.* 30242 dollars.

26. A drover bought 3245 sheep, and sold 1249 of them how many sheep had he left?

27. A general before commencing a battle had 18765 men in his army; after the battle he had only 8530; how many men did he lose? *Ans.* 10235.

28. Two persons bought a block of buildings for 69524 dollars; one paid 47321 dollars; how much did the other pay? *Ans.* 22203 dollars.

29. If a man's annual income is 13460 dollars, and his expenses are 3340 dollars, how much does he save?

Ans. 10120 dollars.

CASE II.

48. When any figure in the subtrahend is greater than the corresponding figure in the minuend.

1. From 846 take 359.

OPERATION.

hunds.	tens.	units.
8	4	6
3	5	9
4	8	7

ANALYSIS. Since we cannot take 9 units from 6 units, we add 10 units to 6 units, making 16 units; and 9 units from 16 units leave 7 units. But as we have added 10 units, or 1 ten to the minuend, we shall have a remainder 1 ten too large, to avoid which, we add 1 ten to the 5 tens in the subtrahend, making 6 tens. We can not take 6 tens from 4 tens; so we add 10 tens to 4, making 14 tens; 6 tens from 14 tens leave 8 tens. Now, having added 10 tens, or 1 hundred, to the minuend, we shall have a remainder 1 hundred too large, unless we add 1 hundred to the 3 hundreds in the subtrahend, making 4 hundreds; 4 hundreds from 8 hundreds leave 4 hundreds, and we have for the total remainder, 487.

NOTE. The process of adding 10 to the minuend is sometimes called *borrowing* 10; and that of adding 1 to the next figure of the subtrahend, *carrying* one.

49. From the preceding example and illustration we have the following general

RULE. I. *Write the less number under the greater, placing units of the same order in the same column.*

II. *Beginning at the right hand, take each figure of the subtrahend from the figure above it, and write the result underneath.*

III. *If any figure in the subtrahend be greater than the corresponding figure above it, add* 10 *to that upper figure before subtracting, and then add* 1 *to the next left hand figure of the subtrahend.*

PROOF. 1st. Add the remainder to the subtrahend; the sum will be equal to the minuend. Or,

2d. Subtract the remainder from the minuend; the difference will be equal to the subtrahend.

EXAMPLES FOR PRACTICE.

	(1.)	(2.)	(3.)	(4.)
Minuend,	753	6731	3248	90361
Subtrahend,	469	2452	1863	6284
Remainder,	284	4279	1385	84077

	(5.) miles.	(6.) bushels.	(7.) dollars.	(8.) feet.
	3146	19472	45268	24760
	2529	14681	24873	3478
	617	4791	20395	21282

	(9.) rods.	(10.) days.	(11.) acres.	(12.) gallons.
	40307	14605	23617	980076
	38421	8341	14309	94087
	1886	6264	9308	885989

	(13.) men.	(14.) sheep.	(15.) barrels.	(16.) tons.
	17380	282731	80014	941000
	3417	90756	43190	5007
	13963	191975	36824	935993

(17.)	(18.)	(19.)
3077097	3000001	1970000
1829164	2199077	1361111
1247933	800924	608889

(20.)	(21.)	(22.)
6000000	8000800	103810040
999999	457776	91300397
5002001	7543024	12509643

23. 234100 — 9970 = how many? *Ans.* 224130.
24. 3749001—349623=how many?
25. 4000320 — 20142 = how many?
26. 14601896 — 764059 = how many?
27. From 4716359 take 2740714. *Ans.* 1975645.
28. From 7867564 take 2948675. *Ans.* 4918889.
29. From 7788996 take 849842. *Ans.* 6939154.
30. From 1073563 take 182000. *Ans.* 891563.
31. From 1111111 take 111112. *Ans.* 999999.
32. Subtract 1234509 from 8643587. *Ans.* 7409078.
33. Subtract 1000 from 1100000. *Ans.* 1099000.
34. Subtract 100701 from 846587.
35. Subtract 432986702100 from 539864298670.
36. Subtract 29176807982 from 86543298765.
37. A speculator bought wild lands for 10580 dollars, and sold them for 7642 dollars; how much did he lose?

Ans. 2938 dollars.

38. Napoleon the Great was born in 1769, and died in 1821; how old was he at his death? *Ans.* 52 years.
39. Gunpowder was invented in 1330, and printing in 1440; how many years between the two? *Ans.* 110.
40. George Washington was born in 1732, and died in 1799; how old was he at his death? *Ans.* 67 years.
41. The first newspaper published in America was issued at Boston in 1704; how long was that before the death of Benjamin Franklin, which occurred in 1790?

Ans. 86 years.

42. The first steamboat in the United States, built by Robert Fulton, in 1807, made a trip from New York to Albany in 33 hours; how many years from that time to the visit of the Great Eastern to this country in 1860?

Ans. 53 years.

43. Queen Victoria was born in 1819; what will be her age in 1862? *Ans.* 43 years.

44. The United States contain 2983153 square miles, and the British North American Provinces 3125401 square miles. How many square miles does the latter country exceed the former? *Ans.* 142248.

EXAMPLES COMBINING ADDITION AND SUBTRACTION.

1. A farmer having 450 sheep, sold 124 at one time, and 96 at another; how many had he left? *Ans.* 230.

2. If a man's income is 175 dollars a month, and he pays 25 dollars for rent, 44 dollars for provisions, and 18 dollars for other expenses, how much will he have left?

Ans. 88 dollars.

3. A man gave his note for 3245 dollars. He paid at one time 780 dollars, and at another 484 dollars; how much remained unpaid? *Ans.* 1981 dollars.

4. A man paid 140 dollars for a horse and 165 dollars for a carriage. He afterward sold them both for 300 dollars; did he gain or lose, and how much? *Ans.* Lost 5 dollars.

5. A flour merchant having 700 barrels of flour on hand, sold 278 barrels to one man, and 142 to another; how many barrels had he left? *Ans.* 280 barrels.

6. Three men bought a farm for 9840 dollars. The first paid 2672 dollars, the second paid 3089 dollars, and the third the remainder; how much did the third pay?

Ans 4079 dollars.

7. A man bought a house for 1500 dollars, and having expended 315 dollars for repairs, sold it for 2000 dollars; how much was his gain? *Ans.* 185 dollars.

8. Henry Jones owns property to the amount of 36748 dollars, of which he has invested in real estate 12850 dollars, in personal property 9086 dollars, and the remainder he has in bank; how much has he in bank? *Ans.* 14812 dollars.

9. A grocer bought 275 pounds of butter of one farmer, and 318 pounds of another; he afterward sold 210 pounds to one customer, and 97 to another; how many pounds had he left? *Ans.* 286 pounds.

10. A man deposited in bank 10476 dollars; he drew out at one time 2356 dollars, at another 1242, and at another 737 dollars; how much had he remaining in bank?

Ans. 6141 dollars.

11. Borrowed of my neighbor at one time 680 dollars, at another time 910 dollars, and at another time 218 dollars. Having paid him 1309 dollars, how much do I still owe him? *Ans.* 499 dollars.

12. A man bought 3 lots; for the first he paid 2480 dollars, for the second 3137 dollars, and for the third as much as for the other two; he afterward sold them for 15000 dollars; how much was his gain? *Ans.* 3766 dollars.

13. A farmer raised 1864 bushels of wheat, and 1129 bushels of corn. Having sold 1340 bushels of wheat, and 1000 bushels of corn, how many bushels of each has he remaining? *Ans.* 524 bushels, and 129 bushels.

14. A gentleman worth 25800 dollars, bequeathed his estate so that each of his two sons should have 9400 dollars, and his daughter the remainder. How much was the daughter's portion?

MULTIPLICATION.

50. **Multiplication** is the process of taking one of two given numbers as many times as there are units in the other.

51. The **Product** is the result obtained.

MULTIPLICATION TABLE.

Once 1 is 1	2 times 1 are 2	3 times 1 are 3	4 times 1 are 4
Once 2 is 2	2 times 2 are 4	3 times 2 are 6	4 times 2 are 8
Once 3 is 3	2 times 3 are 6	3 times 3 are 9	4 times 3 are 12
Once 4 is 4	2 times 4 are 8	3 times 4 are 12	4 times 4 are 16
Once 5 is 5	2 times 5 are 10	3 times 5 are 15	4 times 5 are 20
Once 6 is 6	2 times 6 are 12	3 times 6 are 18	4 times 6 are 24
Once 7 is 7	2 times 7 are 14	3 times 7 are 21	4 times 7 are 28
Once 8 is 8	2 times 8 are 16	3 times 8 are 24	4 times 8 are 32
Once 9 is 9	2 times 9 are 18	3 times 9 are 27	4 times 9 are 36
Once 10 is 10	2 times 10 are 20	3 times 10 are 30	4 times 10 are 40
Once 11 is 11	2 times 11 are 22	3 times 11 are 33	4 times 11 are 44
Once 12 is 12	2 times 12 are 24	3 times 12 are 36	4 times 12 are 48
5 times 1 are 5	6 times 1 are 6	7 times 1 are 7	8 times 1 are 8
5 times 2 are 10	6 times 2 are 12	7 times 2 are 14	8 times 2 are 16
5 times 3 are 15	6 times 3 are 18	7 times 3 are 21	8 times 3 are 24
5 times 4 are 20	6 times 4 are 24	7 times 4 are 28	8 times 4 are 32
5 times 5 are 25	6 times 5 are 30	7 times 5 are 35	8 times 5 are 40
5 times 6 are 30	6 times 6 are 36	7 times 6 are 42	8 times 6 are 48
5 times 7 are 35	6 times 7 are 42	7 times 7 are 49	8 times 7 are 56
5 times 8 are 40	6 times 8 are 48	7 times 8 are 56	8 times 8 are 64
5 times 9 are 45	6 times 9 are 54	7 times 9 are 63	8 times 9 are 72
5 times 10 are 50	6 times 10 are 60	7 times 10 are 70	8 times 10 are 80
5 times 11 are 55	6 times 11 are 66	7 times 11 are 77	8 times 11 are 88
5 times 12 are 60	6 times 12 are 72	7 times 12 are 84	8 times 12 are 96
9 times 1 are 9	10 times 1 are 10	11 times 1 are 11	12 times 1 are 12
9 times 2 are 18	10 times 2 are 20	11 times 2 are 22	12 times 2 are 24
9 times 3 are 27	10 times 3 are 30	11 times 3 are 33	12 times 3 are 36
9 times 4 are 36	10 times 4 are 40	11 times 4 are 44	12 times 4 are 48
9 times 5 are 45	10 times 5 are 50	11 times 5 are 55	12 times 5 are 60
9 times 6 are 54	10 times 6 are 60	11 times 6 are 66	12 times 6 are 72
9 times 7 are 63	10 times 7 are 70	11 times 7 are 77	12 times 7 are 84
9 times 8 are 72	10 times 8 are 80	11 times 8 are 88	12 times 8 are 96
9 times 9 are 81	10 times 9 are 90	11 times 9 are 99	12 times 9 are 108
9 times 10 are 90	10 times 10 are 100	11 times 10 are 110	12 times 10 are 120
9 times 11 are 99	10 times 11 are 110	11 times 11 are 121	12 times 11 are 132
9 times 12 are 108	10 times 12 are 120	11 times 12 are 132	12 times 12 are 144

MENTAL EXERCISES.

1. At 9 cents a pound, what will 7 pounds of sugar cost?

ANALYSIS. Since one pound costs 9 cents, 7 pounds will cost 7 times 9 cents, or 63 cents. Therefore, at 9 cents a pound, 7 pounds of sugar will cost 63 cents.

2. At 6 dollars a week, what will 8 weeks' board cost?

3. When flour is 7 dollars a barrel, what will 11 barrels cost?

4. If Rollin can earn 10 dollars in one month, how much can he earn in 4 months? in 9 months? in 11 months?

5. What will be the cost of 12 pounds of coffee, at 9 cents a pound?

6. At 5 dollars a ton, what will 9 tons of coal cost?

7. At 4 dollars a yard, what will 8 yards of cloth cost?

8. If a pair of boots cost 5 dollars, what will be the cost of 3 pairs? of 6 pairs? of 7 pairs? of 11 pairs?

9. Since 12 inches make a foot, how many inches in 3 feet? in 5 feet? in 7 feet? in 9 feet? in 12 feet?

10. At five cents a quart, what will 6 quarts of milk cost? 10 quarts? 11 quarts?

11. If a man earn 8 dollars in a week, how much can he earn in 6 weeks? in 7 weeks? in 8 weeks? in 9 weeks?

12. If 9 bushels of apples buy one barrel of flour, how many bushels will be required to buy 3 barrels? 5 barrels? 7 barrels? 9 barrels?

13. If 4 men can do a piece of work in 8 days, how many days will it take one man to do it?

14. If 7 men can build a wall in 3 days, how long will it take one man to build it?

15. If a barrel of potatoes last 6 persons 3 weeks, how many weeks will it last one person?

PROMISCUOUS MULTIPLICATION TABLE.

2 times 8 are how many?	2 times 9 are how many?
3 times 9 are how many?	6 times 5 are how many?
4 times 8 are how many?	4 times 7 are how many?
7 times 5 are how many?	9 times 3 are how many?
9 times 4 are how many?	5 times 7 are how many?
6 times 3 are how many?	5 times 8 are how many?
4 times 9 are how many?	9 times 5 are how many?
5 times 9 are how many?	6 times 4 are how many?
7 times 6 are how many?	8 times 3 are how many?

3 times 7 are how many?	7 times 7 are how many?
8 times 9 are how many?	4 times 2 are how many?
6 times 8 are how many?	9 times 9 are how many?
5 times 6 are how many?	4 times 3 are how many?
7 times 3 are how many?	6 times 9 are how many?
6 times 6 are how many?	2 times 6 are how many?
9 times 7 are how many?	8 times 5 are how many?
3 times 8 are how many?	4 times 4 are how many?
7 times 4 are how many?	9 times 8 are how many?

8 times 7 are how many?	2 times 4 are how many?
5 times 4 are how many?	5 times 9 are how many?
3 times 5 are how many?	9 times 8 are how many?
3 times 4 are how many?	3 times 3 are how many?
8 times 6 are how many?	2 times 3 are how many?
7 times 8 are how many?	7 times 4 are how many?
5 times 3 are how many?	0 times 8 are how many?
3 times 6 are how many?	3 times 6 are how many?
8 times 8 are how many?	6 times 10 are how many?

52. The **Multiplicand** is the number to be taken.

53. The **Multiplier** is the number which shows how many times the multiplicand is to be taken.

54. The **Factors** are the multiplicand and multiplier.

55. The **Sign of Multiplication** is the oblique cross, $\times$. It indicates that the numbers connected by it are to be multiplied together; thus $9 \times 6 = 54$, is read 9 times 6 equals 54.

NOTES. 1. Factors are producers, and the multiplicand and multiplier are called factors because they produce the product.

2. Multiplication is a short method of performing addition when the numbers to be added are equal.

CASE I.

56. When the multiplier consists of one figure.

1. Multiply 374 by 6.

OPERATION.

	hunds.	tens.	units.
Multiplicand,	3	7	4
Multiplier,			6
Product,	2	2	4

ANALYSIS. In this example it is required to take 374 six times. If we take the units of each order 6 times, we shall take the entire number 6 times. Therefore, writing the multiplier under the unit figure of the multiplicand, we proceed as follows: 6 times 4 units are 24 units, which is 2 *tens* and 4 *units;* write the 4 units in the product in units' place, and reserve the 2 tens to add to the next product; 6 times 7 tens are 42 tens, and the two tens reserved in the last product added, are 44 tens, which is 4 *hundreds* and 4 *tens;* write the 4 tens in the product in tens' place, and reserve the 4 hundreds to add to the next product; 6 times 3 hundreds are 18 hundreds, and 4 hundreds added are 22 hundreds, which, being written in the product in the places of hundreds and thousands, gives, for the entire product, 2244.

57. The unit value of a number is not changed by repeating the number. As the multiplier always expresses *times*, the product must have the same unit value as the multiplicand. But, since the product of any two numbers will be the same, whichever factor is taken as a multiplier, either factor may be taken for the multiplier or multiplicand.

NOTE. In multiplying, learn to pronounce the partial results, as in addition, without naming the numbers separately. Thus, in the last example, instead of saying 6 times 4 are 24, 6 times 7 are 42 and 2 to carry are 44, 6 times 3 are 18 and 4 to carry are 22; pronounce only the results, 24, 44, 22, performing the operations mentally. This will greatly facilitate the process of multiplying.

EXAMPLES FOR PRACTICE.

	(2.)	(3.)	(4.)	(5.)
Multiplicand,	842	625	718	937
Multiplier,	4	6	7	3
Product,	3368	3750	5026	2811

	(6.)	(7.)	(8.)	(9.)
	4328	5073	1869	3265
	8	5	4	9
	34624	25365	7476	29385

	(10.)	(11.)	(12.)	(13.)
	7186	9010	4079	6394
	3	7	6	8
	21558	63070	24474	51152

(14.)	(15.)	(16.)
340071	760892	1976230
2	4	5
680142	3043568	9881150

17. Multiply 473126 by 9. *Ans.*

18. Multiply 30789167 by 7. *Ans.*

19. Multiply 87231420 by 8. *Ans.*

20. What will be the cost of 9380 bushels of wheat, at 9 shillings a bushel? *Ans.* 84420 shillings.

21. What will be the cost of 4738 tons of coal, at 4 dollars a ton? *Ans.* 18952 dollars.

22. In one mile are 5280 feet; how many feet in 8 miles? *Ans.* 42240 feet.

CASE II.

58. When the multiplier consists of two or more figures.

1. Multiply 746 by 23.

OPERATION.

Multiplicand,	746	
Multiplier,	23	
	2238	3 { times the multiplicand.
	1492	20 { times the multiplicand.
Product,	17158	23 { times the multiplicand.

ANALYSIS. Writing the multiplicand and multiplier as in Case I, we first multiply each figure in the multiplicand by the unit figure of the multiplier, precisely as in Case I. We then multiply by the 2 tens. 2 tens times 6 units, or 6 times 2 tens, are 12 tens, equal to 1 hundred, and 2 tens; we place the 2 tens under the tens figure in the product already obtained, and add the 1 hundred to the next hundreds produced. 2 tens times 4 tens are 8 hundreds, and the 1 hundred of the last product added are 9 hundreds; we write the the 9 in hundreds' place in the product. 2 tens times 7 hundreds are 14 thousands, equal to 1 ten thousand and 4 thousands, which we write in their appropriate places in the the product. Then adding the two products, we have the en tire product, 17158.

Hence we deduce the following general

RULE. I. *Write the multiplier under the multiplicand, placing units of the same order under each other.*

II. *Multiply the multiplicand by each figure of the multiplier successively, beginning with the unit figure, and write the first figure of each partial product under the figure of the multiplier used, writing down and carrying as in addition.*

III. *If there are partial products, add them, and their sum will be the product required.*

PROOF. Multiply the multiplier by the multiplicand, and if the product is the same as the first result, the work is correct.

NOTE. When the multiplier contains two or more figures, the several results obtained by multiplying by each figure are called *partial* products.

EXAMPLES FOR PRACTICE.

(1.)	(2.)	(3.)
34732	56784	34075
14	24	36
138928	227136	204450
34732	113568	102225
486248	1362816	1226700

4. Multiply 177242 by 19. *Ans.* 3367598.
5. Multiply 1429689 by 55.
6. Multiply 364111 by 56. *Ans.* 20390216.
7. Multiply 78540 by 95. *Ans.* 7461300.
8. Multiply 6555 by 39. *Ans.* 255645.
9. Multiply 76419 by 17. *Ans.* 1299123.
10. Multiply 26517 by 45. *Ans.* 1193265.
11. Multiply 108336 by 58. *Ans.* 6283488.
12. Multiply 209402 by 72. *Ans.* 15076944.
13. Multiply 342516 by 56. *Ans.* 19180896.
14. Multiply 764131 by 48. *Ans.* 36678288.

15. There are 52 weeks in a year; how many weeks in 1861 years? *Ans.* 96772 weeks.

16. An army of 5746 men having plundered a city, took so much money that each man received 37 dollars; how much money was taken? *Ans.* 212602 dollars.

17. If it cost 47346 dollars to build one mile of railroad, how much will it cost to build 76 miles?

Ans. 3598296 dollars.

(18.)	(19.)	(20.)
47696	560341	243042
144	304	265
190784	2241364	1215210
190784	1681023	1458252
47696	170343664	486084
6868224		64406130

21. Multiply 45678 by 333. *Ans.* 15210774.
22. Multiply 202842 by 342. *Ans.* 69371964.
23. Multiply 9636799 by 489. *Ans.* 4712394711.
24. Multiply 3064125 by 807. *Ans.* 2472748875.
25. Multiply 5610327 by 2034.
26. Multiply 1900731 by 4006. *Ans.* 7614328386.
27. A gentleman bought 307 horses for shipping, at the rate of 105 dollars each; how much did he pay for the whole?
28. What would be the value of 976 shares of railroad stock, at 98 dollars a share? *Ans.* 95648 dollars.
29. A man bought 48 building lots, at 1236 dollars each; how much did they all cost him? *Ans.* 59328 dollars.
30. How many yards of broadcloth in 487 pieces, each piece containg 37 yards? *Ans.* 18019 yards.
31. If it require 135 tons of iron for one mile of railroad, how many tons will be required for 196 miles?

Ans. 26460.

32. How many oranges in 356 boxes, each box containing 264 oranges? *Ans.* 93984.
33. If it require 6894 shingles for the roof of a house, how many shingles will be required for 19 such houses?

34. 37896×149 = how many? *Ans.* 5646504.
35. 8567×462 = how many? *Ans.* 3957954.
36. 6793×842 = how many? *Ans.* 5719706.
37. 674200×2104 = how many? *Ans.* 1418516800.
38. 15607×3094 = how many? *Ans.* 48288058.
39. 83209×4004 = how many?
40. Multiply 31416 by 175.
41. Multiply 40930 by 779. *Ans.* 31884470.
42. Multiply 4567 by 9009. *Ans.* 41144103.
43. Multiply 7071 by 556. *Ans.* 3931476.
44. Multiply 291042 by 125. *Ans.* 36380250.
45. Multiply 54001 by 5009.
46. Multiply twelve thousand thirteen, by twelve hundred four. *Ans.* 14463652.
47. Multiply thirty-seven thousand seven hundred ninety-six, by four hundred eight.
48. Multiply one million two hundred forty-six thousand eight hnndred fifty-three, by nine thousand seven.

Ans. 11230404971.

49. What will be the cost of building 128 miles of railroad, at 6375 dollars per mile? *Ans.* 816000 dollars.
50. A crop of cotton was put up in 126 bales, each bale containing 572 pounds; what was the weight of the entire crop? *Ans.* 72072 pounds.
51. Two towns, 243 miles apart, are to be connected by a railroad, at a cost of 39760 dollars a mile; how much will be the entire cost of the road? *Ans.* 9661680 dollars.
52. Allowing an acre of land to produce 105 bushels, how much would 246 acres produce? *Ans.* 25830 bushels.
53. If a garrison of soldiers consume 5789 pounds of bread a day, how much will they consume in 287 days?

Ans. 1661443 pounds

CONTRACTIONS.

CASE I.

59. When the multiplier is a composite number.

A **Composite Number** is one that may be produced by multiplying together two or more numbers; thus, 18 is a composite number, since $6\times3=18$; or, $9\times2=18$; or, $3\times3\times2=18$.

60. The **Component Factors** of a number are the several numbers which, multiplied together, produce the given number; thus, the component factors of 20 are 10 and 2, $(10\times2=20)$; or, 4 and 5, $(4\times5=20)$; or, 2 and 2 and 5, $(2\times2\times5=20)$.

NOTE.—The pupil must not confound the *factors* with the *parts* of a number. Thus, the *factors* of which twelve is composed, are 4 and 3, $(4\times3=12)$; while the *parts* of which 12 is composed are 8 and 4, $(8+4=12)$, or 10 and 2, $(10+2=12)$ The *factors* are *multiplied*, while the *parts* are *added*, to produce the number.

1. What will 32 horses cost, at 174 dollars apiece?

OPERATION.		
Multiplicand	174	cost of 1 horse.
1st factor,	4	
	696	cost of 4 horses.
2d factor,	8	
Product,	5568	cost of 32 horses.

ANALYSIS. The factors of 32 are 4 and 8. If we multiply the cost of 1 horse by 4, we obtain the cost of 4 horses; and by multiplying the cost of 4 horses by 8, we obtain the cost of 8 times 4 horses, or 32 horses, the number bought.

61. Hence we have the following

RULE. I. *Separate the composite number into two or more factors.*

II. *Multiply the multiplicand by one of these factors, and that product by another, and so on until all the factors have been used; the last product will be the product required.*

NOTE. The product of any number of factors will be the same in whatever order they are multiplied. Thus, $4\times3\times5=60$, and $5\times4\times3=60$.

EXAMPLES FOR PRACTICE.

1. Multiply 521 by 16=4×4. *Ans.* 8336.
2. Multiply 4350 by 25=5×5. *Ans.* 108750.
3. Multiply 10709 by 36=6×6. *Ans.* 385524.
4. Multiply 21700 by 27=3×9. *Ans.* 585900.
5. Multiply 783473 by 42=6×7. *Ans.* 32905866.
6. Multiply 764131 by 48=6×8. *Ans.* 36678288.
7. Multiply 40567 by 96=8×12. *Ans.* 3894432.
8. Multiply 182642 by 120=4×5×6. *Ans.* 21917040.
9. Multiply 20704 by 84=3×4×7. *Ans.* 1739136.
10. Multiply 564120 by 140=4×5×7. *Ans.* 78976800.
11. What will 56 acres of land cost, at 147 dollars an acre? *Ans.* 8232 dollars.
12. What will 75 yoke of cattle cost, at 184 dollars a yoke? *Ans.* 13800 dollars.
13. If a ship sail 380 miles a day, how far will she sail in 45 days? *Ans.* 17100 miles.
14. What is the value of 3426 pounds of butter, at 18 cents a pound? *Ans.* 61668 cents.
15. What would be the cost of 125 horses, at 208 dollar each? *Ans.* 26000 dollars.
16. What would be the value of 1342 acres of land, at 28 dollars an acre?
17. What will be the cost of 28 pieces of broadcloth, each piece containing 42 yards, at 4 dollars a yard? *Ans.* 4704 dollars.
18. What will be the cost of 16 sacks of coffee, each sack containing 75 pounds, at 9 cents a pound? *Ans.* 10800 cents.

CASE II.

62. When the multiplier is 10, 100, 1000, &c.

If we annex a cipher to the multiplicand, each figure is removed *one* place toward the left, and consequently the value of the whole number is increased ten fold. If two ciphers are annexed, each figure is removed *two* places toward the left, and the value of the number is increased one hundred fold; and every additional cipher increases the value tenfold.

Hence the following

RULE. *Annex as many ciphers to the multiplicand as there are ciphers in the multiplier; the number so formed will be the product required.*

EXAMPLES FOR PRACTICE.

1. Multiply 246 by 10. *Ans.* 2460.
2. Multiply 97 by 100. *Ans.* 9700.
3. Multiply 1476 by 1000. *Ans.* 1476000.
4. Multiply 7361 by 10000. *Ans.* 73610000.
5. At 47 dollars an acre, what will 10 acres of land cost? *Ans.* 470 dollars.
6. What will be the cost of 100 horses, at 95 dollars a head? *Ans.* 9500 dollars.
7. What will be the cost of 1000 fruit trees, at 18 cents apiece? *Ans.* 18000 cents.
8. If one acre of land produce 28 bushels of wheat, how many bushels will 100 acres produce? *Ans.* 2800.
9. If a man save 386 dollars a year, how much will he save in 10 years? *Ans.* 3860 dollars.
10. If the freight on a barrel of flour from Chicago to New York be 47 cents, how much will it be on 100000 barrels? *Ans.* 4700000 cents.

CASE III.

63. When there are ciphers at the right hand of one or both of the factors.

1. Multiply 7200 by 40.

OPERATION.	
Multiplicand,	7200
Multiplier,	40
Product,	288000

ANALYSIS. The multiplicand, factored, is equal to 72×100; the multiplier, factored, is equal to 4×10 and as these factors taken in any order will give the same product, we first multiply 72 by 4, then this product by 100 by annexing two ciphers, and this product by 10 by annexing one cipher. Hence, the following

RULE. *Multiply the significant figures of the multiplicand by those of the multiplier, and to the product annex as many ciphers as there are ciphers on the right of either or both factors.*

EXAMPLES FOR PRACTICE.

	(1.)	(2.)	(3.)
Multiply	3900	1760	37200
By	8000	3500	730000
	31200000	880	1116
		528	2604
		6160000	27156000000

4. Multiply 7030 by 164000. *Ans.* 1152920000.

5. Multiply 27600 by 48000. *Ans.* 1324800000.

6. Multiply 403700 by 30200. *Ans.* 12191740000.

7. At 150 dollars an acre, what will be the cost of 500 acres of land? *Ans.* 75000 dollars.

8. What will be the freight on 4000 barrels of flour, at 50 cents a barrel? *Ans.* 200000 cents.

9. If there are 560 shingles in a bunch, how many shingles in 26000 bunches? *Ans.* 14560000.

EXAMPLES COMBINING ADDITION, SUBTRACTION, AND MULTIPLICATION.

1. Bought 9 cords of wood at 3 dollars a cord, and 15 tons of coal at 5 dollars a ton; what was the cost of the wood and coal? *Ans.* 102 dollars.

2. A grocer bought 6 tubs of butter, each containing 64 pounds, at 14 cents a pound; and 4 cheeses, each weighing 42 pounds, at 8 cents a pound; how much was the cost of the butter and cheese? *Ans.* 6720 cents.

3. If a clerk receive 540 dollars a year salary, and pay 180 dollars for board, 116 dollars for clothing, 58 dollars for books, and 75 dollars for other expenses, how much will he have left at the close of the year? *Ans.* 111 dollars.

4. A farmer having 2150 dollars, bought 536 sheep at 2 dollars a head, and 26 cows at 23 dollars a head; how much money had he left? *Ans.* 480 dollars.

5. A man sold three horses; for the first he received 275 dollars, for the second 87 dollars less than for the first, and for the third as much as for the other two; how much did he receive for the third? *Ans.* 463 dollars.

6. Bought 76 hogs, each weighing 416 pounds, at 7 cents a pound, and sold the same at 9 cents a pound; how much was gained? *Ans.* 63232 cents.

7. A man bought 14 cows at 26 dollars each, 4 horses at 112 dollars each, and 125 sheep at 3 dollars each; he sold the whole for 1237 dollars; did he gain or lose, and how much? *Ans.* Gained 50 dollars.

8. B has 174 sheep, C has three times as many lacking 98, and D has as many as B and C together; how many sheep has D? *Ans.* 598.

9. There are 36 tubs of butter, each weighing 108 pounds; the tubs which contain the butter, each weigh 19

pounds; how much is the weight of the butter without the tubs? *Ans.* 3204 pounds.

10 A man paid for building a house 2376 dollars, and for his farm 4 times as much lacking 970 dollars; how much did he pay for both?

11. A merchant bought 9 hogsheads of sugar at 32 dollars a hogshead, and sold it for 40 dollars a hogshead; how much did he gain? *Ans.* 72 dollars.

12. Bought 360 barrels of flour for 2340 dollars, and sold the same at 8 dollars a barrel; how much was gained by the bargain? *Ans.* 540 dollars.

13. A farmer sold 462 bushels of wheat at 2 dollars a bushel, for which he received 75 barrels of flour at 9 dollars a barrel, and the balance in money; how much money did he receive? *Ans.* 249 dollars.

14. Two persons start from the same point, and travel in opposite directions; one travels at the rate of 28 miles a day, the other at the rate of 37 miles a day; how far apart will they be in 6 days? *Ans.* 390 miles.

15. If a man buy 40 acres of land at 35 dollars an acre, and 56 acres at 29 dollars an acre, and sell the whole for 32 dollars an acre, how much does he gain or lose?

Ans. Gains 48 dollars.

16. In an orchard, 76 apple trees yield 18 bushels of apples each, and 27 others yield 21 bushels each; how much would the apples be worth, at 30 cents a bushel?

Ans. 58050 cents.

17. A man bought two farms, one of 136 acres at 28 dollars an acre, and another of 140 acres at 33 dollars an acre; he paid at one time 4000 dollars, and at another time 1875 dollars; how much remained unpaid?

Ans. 2553 dollars.

DIVISION.

64. **Division** is the process of finding how many times one number is contained in another.

65. The **Quotient** is the result obtained, and shows how many times the divisor is contained in the dividend.

DIVISION TABLE.

1 in 2 2 times	2 in 4 2 times	3 in 6 2 times
1 in 3 3 times	2 in 6 3 times	3 in 9 3 times
1 in 4 4 times	2 in 8 4 times	3 in 12 4 times
1 in 5 5 times	2 in 10 5 times	3 in 15 5 times
1 in 6 6 times	2 in 12 6 times	3 in 18 6 times
1 in 7 7 times	2 in 14 7 times	3 in 21 7 times
1 in 8 8 times	2 in 16 8 times	3 in 24 8 times
1 in 9 9 times	2 in 18 9 times	3 in 27 9 times
4 in 8 2 times	5 in 10 2 times	6 in 12 2 times
4 in 12 3 times	5 in 15 3 times	6 in 18 3 times
4 in 16 4 times	5 in 20 4 times	6 in 24 4 times
4 in 20 5 times	5 in 25 5 times	6 in 30 5 times
4 in 24 6 times	5 in 30 6 times	6 in 36 6 times
4 in 28 7 times	5 in 35 7 times	6 in 42 7 times
4 in 32 8 times	5 in 40 8 times	6 in 48 8 times
4 in 36 9 times	5 in 45 9 times	6 in 54 9 times
7 in 14 2 times	8 in 16 2 times	9 in 18 2 times
7 in 21 3 times	8 in 24 3 times	9 in 27 3 times
7 in 28 4 times	8 in 32 4 times	9 in 36 4 times
7 in 35 5 times	8 in 40 5 times	9 in 45 5 times
7 in 42 6 times	8 in 48 6 times	9 in 54 6 times
7 in 49 7 times	8 in 56 7 times	9 in 63 7 times
7 in 56 8 times	8 in 64 8 times	9 in 72 8 times
7 in 63 9 times	8 in 72 9 times	9 in 81 9 times
10 in 20 2 times	11 in 22 2 times	12 in 24 2 times
10 in 30 3 times	11 in 33 3 times	12 in 36 3 times
10 in 40 4 times	11 in 44 4 times	12 in 48 4 times
10 in 50 5 times	11 in 55 5 times	12 in 60 5 times
10 in 60 6 times	11 in 66 6 times	12 in 72 6 times
10 in 70 7 times	11 in 77 7 times	12 in 84 7 times
10 in 80 8 times	11 in 88 8 times	12 in 96 8 times
10 in 90 9 times	11 in 99 9 times	12 in 108 9 times

MENTAL EXERCISES.

1. How many barrels of flour, at 6 dollars a barrel can be bought for 30 dollars?

ANALYSIS. Since 6 dollars will buy one barrel of flour, 30 dollars will buy as many barrels as 6 dollars, the price of one barrel, is contained times in 30 dollars, which is 5 times. Therefore, at dollars a barrel, 5 barrels of flour can be bought for 30 dollars.

2. How many oranges, at 4 cents apiece, can be bought for 28 cents?

3. How many tons of coal, at 5 dollars a ton, can be bought for 35 dollars?

4. When lard is 7 cents a pound, how many pounds can be bought for 49 cents? for 63 cents? for 84 cents?

5. If a man travel 48 miles in 6 hours, how far does he travel in one hour?

6. At 3 cents apiece, how many lemons can be bought for 24 cents? for 30 cents? for 36 cents?

7. If you give 55 cents to 5 beggars, how many cents do you give to each?

8. If a man build 42 rods of wall in 7 days, how many rods can he build in 1 day?

9. At 4 dollars a cord, how many cords of wood can be bought for 20 dollars? for 28 dollars? for 32 dollars?

10. A farmer paid 33 dollars for some sheep, at 3 dollars apiece; how many did he buy?

11. At 7 cents a pound, how many pounds of sugar can be bought for 63 cents? for 84 cents?

12. If a man spend 5 cents a day for cigars, how many days will 50 cents last him? 60 cents?

13. At 12 cents a pound, how many pounds of coffee can be bought for 48 cents? for 72 cents? for 96 cents? for 120 cents?

PROMISCUOUS DIVISION TABLE.

6 in 36, how many times?	9 in 63, how many times?
7 in 42, how many times?	6 in 12, how many times?
9 in 81, how many times?	7 in 28, how many times?
5 in 35, how many times?	4 in 16, how many times?
8 in 72, how many times?	7 in 49, how many times?
9 in 27, how many times?	4 in 36, how many times?
4 in 20, how many times?	8 in 64, how many times?
6 in 54, how many times?	8 in 40, how many times?

8 in 32, how many times?	4 in 28, how many times?
5 in 45, how many times?	8 in 32, how many times?
6 in 42, how many times?	6 in 48, how many times?
8 in 56, how many times?	9 in 45, how many times?
7 in 63, how many times?	8 in 48, how many times?
3 in 27, how many times?	7 in 56, how many times?
7 in 21, how many times?	3 in 21, how many times?
8 in 16, how many times?	6 in 54, how many times?

4 in 12, how many times?	2 in 16, how many times?
7 in 35, how many times?	4 in 32, how many times?
5 in 10, how many times?	6 in 24, how many times?
7 in 14, how many times?	9 in 72, how many times?
4 in 24, how many times?	5 in 10, how many times?
5 in 30, how many times?	4 in 8, how many times?
9 in 36, how many times?	5 in 20, how many times?
6 in 30, how many times?	2 in 10, how many times?

66. The **Dividend** is the number to be divided.

67. The **Divisor** is the number to divide by.

68. The **Sign of Division** is a short horizontal line, with a point above and one below, $\div$. It indicates that the number before it is to be divided by the number after it. Thus, $20 \div 4 = 5$, is read, 20 divided by 4 is equal to 5.

Division is also expressed by writing the dividend *above*, and the divisor *below*, a short horizontal line;

Thus, $\frac{12}{3} = 4$, shows that 12 divided by 3 equals 4.

CASE I.

69. When the divisor consists of one figure.

1. How many times is 4 contained in 848?

OPERATION.

Dividend, 4)848

Divisor, Quotient, 212

ANALYSIS. After writing the divisor on the left of the dividend, with a line between them, we begin at the left hand and say: 4 is contained in 8, 2 times, and as 8 in the dividend is hundreds, the 2 in the quotient must be hundreds; we therefore write 2 in hundreds' place under the figure divided. 4 is contained in 4, 1 time, and since 4 denotes tens, we write 1 under it in tens' place. 4 in 8, 2 times, and since 8 is units, we write 2 in units' place under it, and we have 212 for the entire quotient.

EXAMPLES FOR PRACTICE.

	(2.)	(3.)	(4.)
Divisor,	3)936 Dividend,	2)4862	4)48844
	312 Quotient.	2431	12211

5. Divide 9963 by 3. *Ans.* 3321.
6. Divide 5555 by 5. *Ans.* 1111.
7. Divide 68242 by 2. *Ans.* 34121.
8. Divide 66666 by 6.

When the divisor is not contained in the *first* figure of the dividend, we find how many times it is contained in the *first two* figures.

9. How many times is 4 contained in 2884?

OPERATION.

4)2884

721

ANALYSIS. As we cannot divide 2 by 4, we say 4 is contained in 28, 7 times, and write the 7 in hundreds' place; then 4 is contained in 8, 2 times, which we write in tens' place under the figure divided; and 4 is contained in 4, 1 time, which we write in units' place in the quotient, and we have the entire quotient, 721.

EXAMPLES FOR PRACTICE.

(10.)	(11.)	(12.)
3)2469	5)3055	2)148624
823	611	74312

13. Divide 4266 by 6. *Ans.* 711.
14. Divide 36488 by 4. *Ans.* 9122.
15. Divide 72999 by 9. *Ans.* 8111.
16. Divide 21777 by 7.

After obtaining the first figure of the quotient, if the divisor is not contained in any figure of the dividend, place a cipher in the quotient, and *prefix* this figure to the next one of the dividend.

NOTE. To *prefix* means to place *before*, or at the *left hand*.

17. How many times is 6 contained in 1824?

OPERATION.

6)1824

304

ANALYSIS. Beginning as in the last examples, we say, 6 is contained in 18, 3 times which we write in hundreds' place in the quotient; then 6 is contained in 2 no times, so we write a cipher (0) in tens' place in the quotient, and prefixing the 2 to the 4, we say 6 is contained in 24, 4 times, which we write in units' place, and we have 304 for the entire quotient.

EXAMPLES FOR PRACTICE.

(18.)	(19.)	(20.)
4)3228	7)28357	3)912246
807	4051	304082

21. Divide 40525 by 5. *Ans.* 8105.
22. Divide 36426 by 6. *Ans.* 6071.
23. Divide 184210 by 2. *Ans.* 92105.
24. Divide 85688 by 8. *Ans.* 10711.
25. Divide 273615 by 3. *Ans.* 91205.

After dividing any figure of the dividend, if there be a remainder, prefix it mentally, to the next figure of the dividend, and then divide this number as before.

31. How many times is 4 contained in 943?

OPERATION.

4)943

235 . . . 3 Rem.

ANALYSIS. Here 4 is contained in 9, 2 times, and there is 1 remainder, which we prefix mentally to the next figure, 4, and say 4 is contained in 14, 3 times, and a remainder of 2, which we prefix to 3, and say, 4 is contained in 23, 5 times, and a remainder of 3. This 3 which is left after performing the last division should be divided by the divisor 4; but the method of doing it cannot be explained here, and so we merely *indicate* the division by placing the divisor under it; thus, $\frac{3}{4}$. The entire quotient is written $235\frac{3}{4}$, which may be read, two hundred thirty-five and *three divided by four*, or, two hundred thirty-five and a *remainder of three.*

NOTE. When the process of dividing is performed mentally, and the results only are written, as in the preceeding examples, the operation is termed *Short Division.*

From the foregoing examples and illustrations, we deduce the following

RULE. I. *Write the divisor at the left of the dividend, with a line between them.*

II. *Beginning at the left hand, divide each figure of the dividend by the divisor, and write the result under the dividend.*

III. *If there be a remainder after dividing any figure, regard it as prefixed to the figure of the next lower order in the dividend, and divide as before.*

IV. *Should any figure or part of the dividend be less than the divisor, write a cipher in the quotient, and prefix the number to the figure of the next lower order in the dividend, and divide as before.*

V. *If there be a remainder after dividing the last figure, place it over the divisor at the right hand of the quotient.*

PROOF. Multiply the divisor and quotient together, and to the product add the remainder, if any; if the result be equal to the dividend, the work is correct.

NOTES. 1. This method of proof depends on the fact that division is the reverse of multiplication. The *dividend* answers to the *product*, the *divisor* to one of the *factors*, and the *quotient* to the *other factor*.

2. In multiplication the two factors are given, to find the product: in division, the product and one of the factors are given, to find the other factor.

EXAMPLES FOR PRACTICE.

1. Divide 8430 by 6.

OPERATION.		PROOF
Divisor. 6)8430	Dividend.	1405 Quotient.
1405	Quotient.	6 Divisor.
		8430 Dividend.

(2.)	(3.)	(4.)
5)730490	7)510384	8)6003424
146098	72912	750428

	Quotients.
5. Divide 87647 by 7.	12521.
6. Divide 94328 by 8.	11791.
7. Divide 43272 by 9.	4808.
8. Divide 377424 by 6.	62904.
9. Divide 975216 by 8.	121902.
10. Divide 46375028 by 7.	6625004.
11. Divide 4763025 by 9.	529225.
12. Divide 42005607 by 7.	6000801.
13. Divide 72000450 by 9.	8000050.
14. Divide 97440643 by 8.	12180803/8.
15. Divide 65706313 by 9.	7300701 4/9.
16. Divide 3627089 by 6.	604514 5/6.
17. Divide 4704091 by 7.	672013.

18. Divide 16344 dollars equally among 6 men; how much will each man receive? *Ans.* 2724 dollars.

19. How many barrels of flour, at 7 dollars a barrel, can be bought for 87605 dollars? *Ans.* 12515 barrels.

20. In one week there are 7 days; how many weeks in 23044 days? *Ans.* 3292 weeks.

21. If 5 bushels of wheat make 1 barrel of flour, how many barrels of flour can be made from 314670 bushels? *Ans.* 62934 barrels.

22. By reading 9 pages a day, how many days will be required to read a book through which contains 1161 pages? *Ans.* 129 days.

23. At 4 dollars a yard, how many yards of broadcloth can be bought for 1372 dollars? *Ans.* 343 yards.

24. If a stage goes at the rate of 8 miles an hour, how long will it be in going 1560 miles? *Ans.* 195 hours.

25. There are 3 feet in 1 yard; how many yards in 206175 feet? *Ans.* 68725 yards.

26. Five partners share equally the loss of a ship and cargo, valued at 760315 dollars; how much is each one's share of the loss? *Ans.* 152063 dollars.

27. If a township of 64000 acres be divided equally among 8 persons, how many acres will each receive? *Ans.* 8000 acres.

28. A miller wishes to put 36312 bushels of grain into 6 bins of equal size; how many bushels must each bin contain? *Ans.* 6052 bushels.

29. How many steps of 3 feet each would a man take in walking a mile, or 5280 feet? *Ans.* 1760 steps.

30. A gentleman left his estate, worth 36105 dollars, to be shared equally by his wife and 4 children; how much did each receive? *Ans.* 7221 dollars.

CASE II.

70. When the divisor consists of two or more figures.

NOTE. To illustrate more clearly the method of operation, we will first take an example usually performed by Short Division.

1. How many times is 4 contained in 1504?

OPERATION.

```
4)1504(376
  12
  --
  30
  28
  --
  24
  24
  --
```

ANALYSIS. *First.* We find how many times the divisor 4, is contained in 15, the first partial dividend, which we find to be 3 times, and a remainder. We place this quotient figure at the right hand of the dividend, with a line between them. *Second.* To find the remainder, we multiply the divisor 4, by this quotient figure 3, and place the product 12, under the figures divided. We subtract the product from the figures divided, and have a remainder of 3. *Third.* Bringing down the next figure of the dividend to the right hand of the remainder, we have 30, the second *partial* dividend. Then 4 is contained in 30, 7 times and a remainder. Placing the 7 at the right hand of the last quotient figure, and multiplying the divisor by it, we place the product 28, under the figures last divided, and subtract as before. To the remainder 2, bring down the next figure 4 of the given dividend, and we have 24 for the third *partial* dividend. Then 4 is contained in 24, 6 times. Multiplying and subtracting as before, we find that nothing remains, and we have for the entire quotient 376.

NOTE. When the whole process of division is written out as above, the operation is termed *Long Division*. The *principle* however is the *same* as Short Division.

Solve the following examples, by Long Division.

2. Divide 4672 by 8. *Ans.* 584.
3. Divide 97636 by 7. *Ans.* 13948.
4. Divide 37863 by 9. *Ans.* 4207.
5. Divide 394064 by 11. *Ans.* 35824.

6. How many times is 23 contained in 17158?

OPERATION.

```
23)17158(746
  161
  ---
  105
   92
  ---
  138
  138
  ---
```

ANALYSIS. As 23 is not contained in the first *two* figures of the dividend, we find how many times it is contained in 171, as the first partial dividend. 23 is contained in 171, 7 times, which we place in the quotient on the right of the dividend. We then multiply the divisor 23, by the quotient figure 7, and subtract the product 161, from the part of the dividend used, and we have a remainder of 10. To this remainder we bring down the next figure of the dividend, making 105 for the second partial dividend. Then, 23 is contained in 105, 4 times, which we place in the quotient. Multiplying and subtracting as before, we have a remainder of 13, to which we bring down the next figure of the dividend, making 138 for the third partial dividend. 23 is contained in 138, 6 times; multiplying and subtracting as before, nothing remains, and we have for the entire quotient, 746.

From the preceding illustrations we derive the following general

RULE. I. *Write the divisor at the left of the dividend, as in short division.*

II. *Divide the least number of the left hand figures in the dividend that will contain the divisor one or more times, and place the quotient at the right of the dividend, with a line between them.*

III. *Multiply the divisor by this quotient figure, subtract the product from the partial dividend used, and to the remainder bring down the next figure of the dividend.*

IV. *Divide as before, until all the figures of the dividend have been brought down and divided.*

V. *If any partial dividend will not contain the divisor,*

place a cipher in the quotient, and bring down the next figure of the dividend, and divide as before.

VI. *If there be a remainder after dividing all the figures of the dividend, it must be written in the quotient, with the divisor underneath.*

NOTES. 1. If any remainder be *equal to*, or *greater* than the divisor, the quotient figure is too *small*, and must be increased.

2. If the product of the divisor by the quotient figure be *greater* than the partial dividend, the quotient figure is too *large*, and must be diminished.

PROOF. The same as in short division.

71. The operations in long division consist of five principal steps, viz.:—

1st. Write down the numbers.
2d. Find how many times.
3d. Multiply.
4th. Subtract.
5th. Bring down another figure.

EXAMPLES FOR PRACTICE.

7. Find how many times 18 is contained in 36838.

OPERATION.

Dividend. Quotient.
Divisor. 18)36838(2046$\frac{10}{18}$
36

83
72

118
108
---
10 Remainder

PROOF.

2046 Quotient.
18 Divisor.

16368
2046

36828
10 Remainder.

36838 Dividend.

8. Divide 79638 by 36.

OPERATION.

36)79638(2212$\frac{6}{36}$
72

76
72

43
36

78
72
---
6 Rem.

9. Divide 93975 by 84.

OPERATION.

84)93975(1118$\frac{63}{84}$
84
--
99
84

157
84

735
672
---
63 Rem.

10. Divide 408722 by 136.

OPERATION.

136)408722(3005
408

722
680
---
42 Rem.

11. Divide 104762 by 109.

OPERATION.

109)104762(961
981

666
654

122
109
---
13 Rem.

12. Divide 178464 by 16. *Ans.* 11154.
13. Divide 15341 by 29. *Ans.* 529.
14. Divide 463554 by 39. *Ans.* 11886.
15. Divide 1299123 by 17. *Ans.* 76419.
16. Divide 161700 by 15. *Ans.* 10780.
17. Divide 47653 by 24.
18. Divide 765431 by 42.

	Quotients.	Rem.
19. Divide 6783 by 15.	452	3.
20. Divide 7831 by 18.	435	1.
21. Divide 9767 by 22.	443	21.
22. Divide 7654 by 24.	318	22.
23. Divide 767500 by 23.	33369	13.
24. Divide 250765 by 34.	7375	15.
25. Divide 5571489 by 43.	129569	22.
26. Divide 153598 by 29.	5296	14.
27. Divide 301147 by 63.	4780	7.
28. Divide 40231 by 75.	536	31.
29. Divide 52761878 by 126.	418745	8.
30. Divide 92550 by 25.	3702	
31. Divide 7461300 by 95.	78540	
32. Divide 1193288 by 45.	26517	23.
33. Divide 5973467 by 243.	24582	41.
34. Divide 69372168 by 342.	202842	204.
35. Divide 863256 by 736.	1172	664.
36. Divide 1893312 by 912.	2076	
37. Divide 833382 by 207.	4026	
38. Divide 52847241 by 607.	87063	
39. Divide 13699840 by 342.	40058	4.
40. Divide 946656 by 1038.	912	
41. Divide 46447786 by 1234.	37640	26.
42. Divide 28101418481 by 1107.	25385201	974.
43. Divide 48288058 by 3094.	15607	
44. Divide 47254149 by 4674.	10110	9.

45. A man bought 114 acres of land for 4104 dollars, what was the average price per acre? *Ans.* 36 dollars.

46. Nine thousand dollars was paid to 75 operatives: how much did each receive? *Ans.* 120 dollars.

47. There are 24 hours in a day; how many days in 11424 hours? *Ans.* 476.

48. In one hogshead are 63 gallons; how many hogsheads in 6615 gallons? *Ans.* 105.

49. If a man travel 48 miles a day, how long will it take him to travel 1296 miles? *Ans.* 27 days.

50. If a person can count 8677 in an hour, how long will it take him to count 38369694? *Ans.* 4422 hours.

51. If it cost 5987520 dollars to construct a railroad 576 miles long, what will be the average cost per mile?

Ans. 10395 dollars.

52. The Memphis and Charleston railroad is 287 miles in length, and cost 5572470 dollars; what was the average cost per mile? *Ans.* $19416\frac{78}{287}$ dollars.

53. A garrison consumed 1712 barrels of flour in 107 days; how much was that per day? *Ans.* 16 barrels.

54. How long would it take a vessel to sail from New York to China, allowing the distance to be 9072 miles, and the ship to sail 144 miles a day? *Ans.* 63 days.

55. How long could 27 men subsist on a stock of provision, that would last 1 man 3456 days? *Ans.* 128 days.

56. A drover received 10362 dollars, for 314 head of cattle; how much was their average value per head?

Ans. 33 dollars.

57. If 42864 pounds of cotton be packed in 94 bales, what is the average weight of each bale? *Ans.* 456 pounds.

58. If a field containing 42 acres produce 1659 bushels of wheat, what will be the number of bushels per acre?

Ans. $39\frac{2}{4}\frac{1}{2}$ bushels.

59. In what time will a reservoir containing 109440 gallons, be emptied by a pump discharging 608 gallons per hour? *Ans.* 180 hours.

CONTRACTIONS.

CASE I.

72. When the divisor is 10, 100, 1000, &c.

1. Divide 374 by 10.

OPERATION.

1|0)37|4

Quotient, 37---4 Rem.

or, $37\frac{4}{10}$, *Ans.*

ANALYSIS. Since we have shown, that to remove a figure one place toward the left by annexing a cipher increases its value tenfold, or multiplies it by 10, so, on the contrary, by cutting off or taking away the right hand figure of a number, each of the other figures is removed one place toward the right, and, consequently, the value of each is diminished tenfold, or divided by 10.

For similar reasons, if we cut off *two* figures, we divide by 100, if *three*, we divide by 1000, and so on. Hence the

RULE. *From the right hand of the dividend cut off as many figures as there are ciphers in the divisor. Under the figures so cut off, place the divisor, and the whole will form the quotient.*

EXAMPLES FOR PRACTICE.

	Quotients.	Rem's.
2. Divide 13705 by 100.	137	5.
3. Divide 50670 by 100.	506	70.
4. Divide 320762 by 1000.	320	762.
5. Divide 14030731 by 10000.	1403	731.
6. Divide 9021300640 by 100000.	90213	640.

7. A man sold 100 acres of land for 3725 dollars; how much did he receive an acre? *Ans.* $37\frac{25}{100}$ dollars.

8. Bought 1000 barrels of flour for 6080 dollars; how much did it cost me a barrel? *Ans.* $6\frac{80}{1000}$ dollars.

9. Paid 12560 dollars for 10000 bushels of wheat; how much was the cost per bushel? *Ans.* $1\frac{2560}{10000}$ dollars.

CASE III.

73. When there are ciphers on the right hand of the divisor.

1. Divide 437661 by 800.

OPERATION.

8|00)4376|61

547---61 Rem.

ANALYSIS. In this example we resolve 800 into the factors 8 and 100, and divide first by 100, by cutting off two right hand figures of the dividend, and we have a quotient of 4376, and a remainder of 61. We next divide by 8, and obtain 547 for a quotient; and the entire quotient is $547\frac{61}{800}$.

2. Divide 34716 by 900.

OPERATION.

9|00)347|16

Quotient, 38--516 Rem.

or, $38\frac{516}{900}$, *Ans.*

ANALYSIS. Dividing as in the last example, we have a quotient of 38, and a remainder of 5 after dividing by 9, which we prefix to the figures cut off from the dividend, making a true remainder of 516, and the entire quotient $38\frac{516}{900}$. Hence

RULE. I. *Cut off the ciphers from the right of the divisor, and the same number of figures from the right of the dividend.*

II. *Divide the remaining figures of the dividend by the remaining figures of the divisior, and the result will be the quotient. If there be a remainder after this division, prefix it to the figures cut off from the dividend, and this will form the true remainder.*

EXAMPLES FOR PRACTICE.

	Quotients.	Rem's.
3. Divide 46820 by 400.	117	20.
4. Divide 130627 by 800.	163	227.
5. Divide 76173 by 320.	238	13.
6 Divide 378000 by 1200.	315	

7. Divide 674321 by 11200.	60	2321.
8. Divide 64613214 by 4000.	16153	1214.
9. Divide 146200 by 430.	340	
10. Divide 7380964 by 23000.	320	20964.
11. Divide 58677000 by 1800.	32598	600.

EXAMPLES IN THE PRECEDING RULES.

1. A speculator bought at different times 320 acres, 175 acres, 87 acres, and 32 acres of land, and afterward sold 467 acres; how many acres had he left? *Ans.* 147 acres.

2. Two men travel in opposite directions; one travels 31 miles a day, the other 43 miles a day; how far apart will they be in 12 days? *Ans.* 888 miles.

3. A tobacconist has 6324 pounds of tobacco, which he wishes to pack in boxes containing 62 pounds each; how many boxes must he procure to contain it? *Ans.* 102.

4. A farmer sold 15 tons of hay at 9 dollars a ton, and 25 cords of wood at 4 dollars a cord, and wished to divide the amount equally among 5 creditors; how much would each receive? *Ans.* 47 dollars.

5. If you deposit 216 cents each week in a savings bank, and take out 89 cents a week, how many cents will you have in bank at the end of 36 weeks? *Ans.* 4572 cents.

6. The product of two numbers is 8928, and one of the numbers is 72; what is the other number? *Ans.* 124.

7. The dividend is 7280, and the quotient is 208; what is the divisor? *Ans.* 35.

8. What is the remainder after dividing 876437 by 16900? *Ans.* 14537.

9. A man sold 6 horses at 125 dollars each, 25 head of cattle at 30 dollars each, and with the proceeds bought land at 25 dollars an acre; how many acres did he buy?

Ans. 60 acres.

10. If a mechanic receives 784 dollars a year for labor, and his expenses are 426 dollars a year, how much can he save in 6 years? *Ans.* 2148 dollars.

11. A farmer sold 40 bushels of wheat at 2 dollars a bushel, and 16 cords of wood at 3 dollars a cord. He received 15 yards of cloth at 4 dollars a yard, and the remainder in money; how much money did he receive? *Ans.* 68 dollars.

12. How many pounds of cheese worth 10 cents a pound, can be bought for 22 pounds of butter worth 15 cents a pound? *Ans.* 33 pounds.

13. If 56 yards of cloth cost 336 dollars, how much will 12 yards cost, at the same rate? *Ans.* 72 dollars.

14. If 100 barrels of flour cost 600 dollars, what will 350 barrels cost, at the same rate? *Ans.* 2100 dollars.

15. How long can 60 men subsist on an amount of food that will last 1 man 7620 days? *Ans.* 127 days.

16. If I buy 225 barrels of flour for 1125 dollars, and sell the same for 1800 dollars, how much do I gain on each barrel? *Ans.* 3 dollars.

17. A man sold his house and lot for 5670 dollars, and took his pay in bank stock at 90 dollars a share; how many shares did he receive? *Ans.* 63 shares.

18. How many pounds of tea worth 75 cents a pound, ought a man to receive in exchange for 27 bushels of oats, worth 50 cents a bushel? *Ans.* 18 pounds.

19. The quotient of one number divided by another is 40, the divisor is 364, and the remainder 120; what is the dividend? *Ans.* 14680.

20. How many tons of hay at 12 dollars a ton, must be given for 21 cows at 24 dollars apiece? *Ans.* 42 tons.

21. Bought 150 barrels of flour for 1050 dollars, and sold 107 barrels of it at 9 dollars a barrel, and the remainder at 7 dollars a barrel; did I gain or lose, and how much?

Ans. gained 214 dollars.

22. A mechanic earns 45 dollars a month, and his necessary expenses are 27 dollars a month. How long will it take him to pay for a farm of 50 acres, at 27 dollars an acre? *Ans.* 75 months.

23. How many barrels of flour at 7 dollars a barrel, will pay for 30 tons of coal, at 4 dollars a ton, and 44 cords of wood, at 3 dollars a cord? *Ans.* 36 barrels.

PROBLEMS IN SIMPLE INTEGRAL NUMBERS.

74. The four operations that have now been considered, viz., Addition, Subtraction, Multiplication, and Division, are all the operations that can be performed upon numbers, and hence they are called the *Fundamental Rules*.

75. In all cases, the numbers operated upon and the results obtained, sustain to each other the relation of a whole to its parts. Thus,

I. *In Addition*, the numbers added are the parts, and the sum or amount is the whole.

II. *In Subtraction*, the subtrahend and remainder are the parts, and the minuend is the whole.

III. *In Multiplication*, the multiplicand denotes the value of one part, the multiplier the number of parts, and the product the total value of the whole number of parts.

IV. *In Division*, the dividend denotes the total value of the whole number of parts, the divisor the value of one part, and the quotient the number of parts; or the divisor the number of parts, and the quotient the value of one part.

76. Let the pupil be required to illustrate the following problems by original examples.

Problem 1. Given, several numbers, to find their sum.

Prob. 2. Given, the sum of several numbers and all of them but one, to find that one.

Prob. 3. Given, two numbers, to find their difference.

Prob. 4. Given, the minuend and subtrahend, to find the remainder.

Prob. 5. Given, the minuend and remainder, to find the subtrahend.

Prob. 6. Given, the subtrahend and remainder, to find the minuend.

Prob. 7. Given, two or more numbers, to find their product.

Prob. 8. Given, the multiplicand and multiplier, to find the product.

Prob. 9. Given, the product and multiplicand, to find the multiplier.

Prob. 10. Given, the product and multiplier, to find the multiplicand.

Prob. 11. Given, two numbers, to find their quotients.

Prob. 12. Given, the divisor and dividend, to find the quotient.

Prob. 13. Given, the divisor and quotient, to find the dividend.

Prob. 14. Given, the dividend and quotient, to find the divisor.

Prob. 15. Given, the divisor, quotient, and remainder, to find the dividend.

Prob. 16. Given, the dividend, quotient, and remainder to find the divisor.

FRACTIONS.

DEFINITIONS, NOTATION, AND NUMERATION.

77. If a unit be divided into 2 equal parts, one of the parts is called *one half.*

If a unit be divided into 3 equal parts, one of the parts is called *one third,* two of the parts *two thirds.*

If a unit be divided into 4 equal parts, one of the parts is called *one fourth,* two of the parts *two fourths,* three of the parts *three fourths.*

If a unit be divided into 5 equal parts, one of the parts is called *one fifth,* two of the parts *two fifths,* three of the parts *three fifths,* &c.

And since *one half, one third, one fourth,* and all other *equal parts* of an *integer* or *whole thing,* are each in themselves *entire* and complete, the *parts of a unit* thus used are called *fractional units;* and the numbers formed from them, *fractional numbers.* Hence

78. A **Fractional Unit** is one of the equal parts of an integral unit.

79. A **Fraction** is a fractional unit, or a collection of fractional units.

80. Fractional units take their *name,* and their *value,* from the *number* of parts into which the integral unit is divided. Thus, if we divide an orange into 2 equal parts, the parts are called *halves;* if into 3 equal parts, *thirds;* if into 4 equal parts, *fourths,* &c.; and each *third* is less in value than each *half,* and each *fourth* less than each *third;* and the greater the *number* of parts, the less their *value.*

The parts of a fraction are expressed by figures; thus,

One half	is written	$\frac{1}{2}$	One fifth	is written	$\frac{1}{5}$
One third	"	$\frac{1}{3}$	Two fifths	"	$\frac{2}{5}$
Two thirds	"	$\frac{2}{3}$	One seventh	"	$\frac{1}{7}$
One fourth	"	$\frac{1}{4}$	Three eighths	"	$\frac{3}{8}$
Two fourths	"	$\frac{2}{4}$	Five ninths	"	$\frac{5}{9}$
Three fourths	"	$\frac{3}{4}$	Eight tenths	"	$\frac{8}{10}$

To write a fraction, therefore, two integers are required, one written above the other with a line between them.

81. The **Denominator** of a fraction is the number below the line. It shows into how many parts the integer or unit is divided, and determines the *value* of the *fractional unit.*

82. The **Numerator** is the number above the line. It numbers the *fractional units,* and shows how many are taken.

83. Thus, if one dollar be divided into 4 equal parts, the parts are called *fourths,* the fractional unit being *one fourth,* and three of these parts are called *three fourths* of a dollar, and may be written

$\frac{3}{4}$ — 3 the number of parts or *fractional units* taken; 4 the number of parts or *fractional units* into which the dollar is divided.

84. The **Terms** of a fraction are the numerator and denominator, taken together.

85. *Fractions indicate division,* the numerator answering to the dividend, and the denominator to the divisor. Hence,

86. The **Value** of a fraction is the quotient of the numerator divided by the denominator.

Thus; the quotient of 4 divided by 5 is $\frac{4}{5}$, or $\frac{4}{5}$ expresses the quotient of which $\left\{\begin{array}{l}4 \text{ is the dividend.}\\ 5 \text{ is the divisor.}\end{array}\right.$

1. What is 1 half of 8?

ANALYSIS. It is the quotient of 8 divided by 2, which is 4; or, it is a number, which taken 2 times, will make 8, which is 4. Therefore, 4 is 1 half of 8.

2. What is 2 thirds of 9?

ANALYSIS. Since 1 third of 9 is 3, 2 thirds of 9 is 2 times 3, which is 6. Therefore, 2 thirds of 9 is 6.

Hence, to obtain *one half*, *one third*, *one fourth*, or *any fractional part* of a number, we divide that number by the *denominator of the fraction expressing the parts*; and to obtain any given number of such parts, we *multiply that part by the number of parts expressed by the numerator of the same fraction.*

3. What is 1 fourth of 12? 3 fourths of 12?

4. What is 1 fifth of 20? 3 fifths? 4 fifths?

5. What is 1 eight of 40? 3 eighths? 5 eighths?

6. What is 2 sevenths of 21? 5 sevenths of 35? 6 sevenths of 49?

7. What is 1 ninth of 63? 2 ninths of 27? 4 ninths of 36? 5 ninths of 45? 7 ninths of 81?

8. What is 1 twelfth of 48? 5 twelfths? 7 twelfths?

9. If a pound of coffee cost 15 cents, how much will 1 third of a pound cost? 2 thirds?

10. A farmer having 60 sheep, sold 1 fifth of them to one man, and 3 fifths to another; how many did he sell to both?

11. A boy having 48 cents, spent 5 eighths of them; how many had he left?

12. Paid 108 dollars for a horse, and 9 twelfths as much for a carriage; how much did the carriage cost?

13. William had 120 pennies, and James had 7 tenths as many; how many had James?

87. It is often required to express by a fraction, what part one number is ot another number.

1. What part of 5 is 3?

ANALYSIS. Since 1 is 1 fifth of 5, 3 must be 3 times 1 fifth of 5, or 3 fifths of 5. Therefore, 3 is 3 fifths of 5.

NOTE. The number preceded by the word *of* is generally made the denominator or divisor, and the other number called the part, the numerator or dividend.

2. What part of 6 is 3? 4? 5? 1?

3. What part of 9 is 2? 3? 5? 6? 1? 4?

4. What part of 10 is 7? 6? 3? 1? 9? 8? 4?

5. What part of 12 is 3? 5? 6? 8? 9? 7? 10? 11?

6. What part of 14 is 5? 7? 9? 3? 6? 11? 8? 15?

7. What part of 15 bushels, is 3 bushels? 7 bushels? 9 bushels 11 bushels?

8. What part of 18 dollars, is 7 dollars? 5 dollars? 9 dollars? 17 dollars?

9. If 6 oranges cost 30 cents, what part of 30 cents will 1 orange cost? 2 oranges? 3 oranges? 5 oranges?

EXAMPLES IN WRITING AND READING FRACTIONS.

Express the following fractions by figures:—

1. Nine *twelfths*. *Ans.* $\frac{9}{12}$.
2. Eleven *fifteenths*. *Ans.* $\frac{11}{15}$.
3. Twenty-four *forty-ninths*. *Ans.* $\frac{24}{49}$.
4. Forty-four *sixty-ninths*. *Ans.* $\frac{44}{69}$.
5. One hundred twenty, *four hundred fiftieths*.

Read the the following fractions:

6. $\frac{7}{15}$, $\frac{12}{40}$, $\frac{27}{56}$, $\frac{84}{120}$, $\frac{142}{347}$, $\frac{15}{869}$, $\frac{156}{90}$, $\frac{459}{317}$.

7. If the fractional unit is 28, express 9 fractional units, 16, 17; 22; 27.

8. If the fractional unit is 96, express 27 fractional units; 42; 75

88. Fractions are distinguished as *Proper* and *Improper*.

A **Proper Fraction** is one whose numerator is less than its denominator; its value is less than the unit, 1. Thus, $\frac{7}{12}$, $\frac{5}{16}$, $\frac{9}{10}$, $\frac{72}{84}$ are proper fractions.

An **Improper Fraction** is one whose numerator equals or exceeds its denominator; its value is never less than the unit, 1. Thus, $\frac{7}{7}$, $\frac{3}{3}$, $\frac{15}{4}$, $\frac{35}{8}$, $\frac{50}{10}$, $\frac{180}{90}$ are improper fractions.

89. A **Mixed Number** is a number expressed by an integer and a fraction; thus, $4\frac{1}{4}$, $17\frac{8}{25}$, $9\frac{3}{10}$ are mixed numbers.

REDUCTION.

90. The **Reduction** of a fraction is the process of changing its terms, or its form, without altering its value.

CASE I.

91. To reduce fractions to their lowest terms.

A fraction is in its *lowest terms* when no number greater than 1 will exactly divide both numerator and denominator without a remainder.

1. Reduce $\frac{2}{4}$ to its lowest terms.

ANALYSIS. It is plain, that the numerator 2, and the denominator 4, are both divisible by 2, without remainders; hence

$$\frac{2\div 2}{4\div 2}=\frac{1}{2}$$

The terms thus obtained, viz., 1, the numerator, and 2, the denominator, are not divisible by any number larger than 1, and therefore are the *smallest* terms by which the value of $\frac{2}{4}$ can be expressed.

2. Reduce $\frac{3}{6}$ to its lowest terms.
3. Reduce $\frac{5}{10}$ to its lowest terms.
4. Reduce $\frac{8}{6}$ to its lowest ferms.
5. Reduce $\frac{5}{20}$ to its lowest terms.
6. Reduce $\frac{10}{30}$ to its lowest terms.

7. Reduce $\frac{48}{60}$ to its lowest terms.

OPERATION.

$2)\frac{48}{60}=\frac{24}{30}$; $2)\frac{24}{30}=\frac{12}{15}$;
$3)\frac{12}{15}=\frac{4}{5}$ *Ans.*
or, $12)\frac{48}{60}=\frac{4}{5}$ *Ans.*

ANALYSIS. Dividing both terms of a fraction by the same number does not alter the value of the fraction or quotient; hence, we divide both terms of $\frac{48}{60}$ by 2, and obtain $\frac{24}{30}$; dividing both terms of this fraction by 2, we have $\frac{12}{15}$ as the result; finally, dividing the terms of this fraction by 3, we have $\frac{4}{5}$, and as no number greater than 1 will divide the terms of this fraction without a remainder, $\frac{4}{5}$ are the lowest terms in which the value of $\frac{48}{60}$ can be expressed. We may obtain the final result more readily, by dividing the terms of this fraction by the largest number that will divide both without a remainder, as in the above example; if we divide by 12, we obtain $\frac{4}{5}$, the answer. Hence the

RULE. *Divide the terms of the fraction by any number greater than* 1, *that will divide both without a remainder, and the quotients obtained in the same manner, until no number greater than* 1 *will so divide them; the last quotients will be the lowest terms of the given fraction.*

EXAMPLES FOR PRACTICE.

8. Reduce $\frac{60}{72}$ to its lowest terms. *Ans.* $\frac{5}{6}$.
9. Reduce $\frac{72}{120}$ to its lowest terms. *Ans.* $\frac{3}{5}$.
10. Reduce $\frac{98}{112}$ to its lowest terms. *Ans.* $\frac{7}{8}$.
11. Reduce $\frac{126}{162}$ to its lowest terms. *Ans.* $\frac{7}{9}$.
12. Reduce $\frac{144}{156}$ to its lowest terms. *Ans.* $\frac{12}{13}$.
13. Reduce $\frac{240}{322}$ to its lowest terms. *Ans.* $\frac{120}{161}$.
14. Reduce $\frac{216}{288}$ to its lowest terms. *Ans.* $\frac{3}{4}$.
15. Reduce $\frac{45}{315}$ to its lowest terms. *Ans.* $\frac{1}{7}$.
16. Reduce $\frac{825}{960}$ to its lowest terms. *Ans.* $\frac{55}{64}$.
17. Reduce $\frac{32}{256}$ to its lowest terms. *Ans.* $\frac{1}{8}$.
18. Reduce $\frac{171}{180}$ to its lowest terms. *Ans.* $\frac{19}{20}$.
19. Reduce $\frac{56000}{75553}$ to its lowest terms. *Ans.* $\frac{11300}{15111}$.

CASE II.

92. To change an improper fraction to a whole or mixed number.

1. In $\frac{12}{4}$ how many times 1?

ANALYSIS. Since 1 equals $\frac{4}{4}$, $\frac{12}{4}$ equal as many times 1, as $\frac{4}{4}$ are contained times in $\frac{12}{4}$, which are 3 times. Therefore, $\frac{12}{4}$ are 3 times 1, or 3.

2. How many times 1 in $\frac{15}{3}$? in $\frac{18}{6}$? in $\frac{20}{5}$?
3. How many times 1 in $\frac{28}{7}$? in $\frac{24}{6}$? in $\frac{32}{8}$?
4. How many times 1 in $\frac{64}{8}$? in $\frac{50}{10}$? in $\frac{48}{6}$?
5. How many times 1 in $\frac{72}{9}$? in $\frac{66}{11}$? in $\frac{96}{12}$?

NOTE. When the denominator is not an exact divisor of the numerator, the result will be a mixed number.

6. In $\frac{16}{7}$ how many times 1?

OPERATION.

7)16

$2\frac{2}{7}$ *Ans.*

ANALYSIS. Since 1 equals $\frac{7}{7}$, $\frac{16}{7}$ equal as many times 1 as 7 is contained times in 16, which is $2\frac{2}{7}$ times. Hence the

RULE. *Divide the numerator by the denominator.*

EXAMPLES FOR PRACTICE.

7. In $\frac{196}{8}$ how many times 1? *Ans.* $24\frac{1}{2}$.
8. In $\frac{228}{12}$ of a year how many years? *Ans.* 19.
9. In $\frac{1284}{12}$ of a pound how many pounds? *Ans.* 107.
10. In $\frac{672}{112}$ of a mile how many miles? *Ans.* 6.
11. In $\frac{7007}{33}$ of a rod how many rods? *Ans.* $212\frac{1}{3}$.
12. In $\frac{22455}{10}$ of a dollar how many dollars?
13. Reduce $\frac{96}{16}$ to a whole number. *Ans.* 6.
14. Reduce $\frac{140}{25}$ to a mixed number. *Ans.* $5\frac{3}{5}$.
15. Reduce $\frac{3224}{18}$ to a whole number. *Ans.* 18.
16. Reduce $\frac{14566}{24}$ to a mixed number. *Ans.* $60\frac{2}{3}$.
17. Change $3\frac{246}{48}$ to a mixed number. $67\frac{5}{8}$.
18. Change $2\frac{184}{42}$ to a whole number. *Ans.* 52.

CASE III.

93. To reduce a whole or mixed number to an improper fraction.

1. How many thirds in 4?

ANALYSIS. Since in 1 there are 3 *thirds*, in 4 there are 4 times 3 *thirds*, or 12 thirds. Therefore, there are $\frac{12}{3}$ in 4.

2. How many *fourths* in 2? in 3? in 5?
3. How many *halves* in 5? in 7? in 8? in 9?
4. How many *sixths* in 3? in 5? in 7? in 10?
5. How many *tenths* in 4? in 8? in 9? in 6?
6. How many *fifths* in 2 whole oranges? in 4? in 5?
7. How many *eighths* in 4 whole dollars? in 5? in 6?
8. In $3\frac{5}{8}$ dollars how many *eighths* of a dollar?

OPERATION.

$3\frac{5}{8}$
8
—
$24+5=\frac{29}{8}$

ANALYSIS. Since in 1 dollar there are 8 *eighths*, in 3 dollars there are 3 times 8 eighths, or 24 eighths, and 5 eighths added, make $\frac{29}{8}$.

RULE. *Multiply the whole number by the denominator of the fraction; to the product add the numerator, and under the result write the denominator.*

EXAMPLES FOR PRACTICE.

9. Reduce $6\frac{3}{4}$ to an improper fraction. *Ans.* $\frac{27}{4}$.
10. Reduce $7\frac{5}{9}$ to an improper fraction. *Ans.* $\frac{68}{9}$.
11. Reduce 15 to a fraction whose denominator is 7. *Ans.* $\frac{105}{7}$.
12. Reduce 120 to *twelfths*. *Ans.* $\frac{1440}{12}$.
13. In $242\frac{2}{3}$ of an acre how many *thirds* of an acre?
14. In $75\frac{7}{8}$ bushels how many *eighths*? *Ans.* $\frac{607}{8}$.
15. In 24 pounds how many *sixteenths*? *Ans.* $\frac{384}{16}$.
16. In 52 weeks how many *sevenths*? *Ans.* $\frac{364}{7}$.
17. Change $14\frac{9}{24}$ to an improper fraction. *Ans.* $\frac{345}{24}$.

CASE IV.

94. To reduce two or more fractions to a common denominator.

A **Common Denominator** is a denominator common to two or more fractions.

NOTE. Any number that can be divided by each of the denominators of the given fractions, may be taken for the common denominator.

1. Reduce $\frac{1}{4}$ and $\frac{2}{3}$ to fractions having a common denominator.

ANALYSIS. 12 is exactly divisible by 4 and 3, and may therefore be taken for a common denominator. Since in 1 there are $\frac{12}{12}$, in $\frac{1}{4}$ of 1 there must be $\frac{1}{4}$ of $\frac{12}{12}$ or $\frac{3}{12}$; and in $\frac{2}{3}$ of 1 there must be $\frac{2}{3}$ of $\frac{12}{12}$, or $\frac{8}{12}$. Therefore $\frac{1}{4}$ and $\frac{2}{3}$ are equal to $\frac{3}{12}$ and $\frac{8}{12}$.

2. Reduce $\frac{3}{4}$ and $\frac{1}{6}$ to a common denominator.
3. Reduce $\frac{1}{3}$ and $\frac{3}{5}$ to a common denominator.
4. Reduce $\frac{1}{4}$ and $\frac{5}{8}$ to a common denominator.
5. Reduce $\frac{5}{6}$ and $\frac{3}{5}$ to a common denominator.

OPERATION.

$5\times5=25$
$\overline{6\times5=30}$

$3\times6=18$
$\overline{5\times6=30}$

ANALYSIS. We multiply the terms of the first fraction $\frac{5}{6}$, by the denominator 5 of the second, and the terms of the fraction $\frac{3}{5}$, by the denominator 6 of the first. This must reduce each fraction to the same denominator 30, for each new denominator will be the product of the given denominators. Hence the

RULE. *Multiply both terms of each fraction by the denominators of all the other fractions.*

NOTE. Mixed numbers must first be reduced to improper fractions.

EXAMPLES FOR PRACTICE.

6. Reduce $\frac{4}{7}$ and $\frac{3}{4}$ to a common denominator.

Ans. $\frac{16}{28}$, $\frac{21}{28}$.

7. Reduce $\frac{4}{5}$ and $\frac{7}{8}$ to a common denominator.

Ans. $\frac{32}{40}$, $\frac{35}{40}$.

8. Reduce $\frac{2}{3}$ and $\frac{6}{7}$ to a common denominator.

Ans. $\frac{14}{21}$, $\frac{18}{21}$.

9. Reduce $\frac{1}{6}$ and $\frac{7}{10}$ to a common denominator.

Ans. $\frac{10}{60}$, $\frac{42}{60}$.

10. Reduce $\frac{3}{8}$ and $\frac{5}{12}$ to a common denominator.

Aus. $\frac{36}{96}$, $\frac{40}{96}$.

11. Reduce $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$ to a common denominator.

Ans. $\frac{12}{24}$, $\frac{16}{24}$, $\frac{18}{24}$.

12. Reduce $\frac{1}{3}$, $\frac{3}{5}$, and $\frac{4}{7}$ to a common denominator.

Ans. $\frac{35}{105}$, $\frac{63}{105}$, $\frac{60}{105}$.

13. Reduce $\frac{4}{5}$, $\frac{1}{6}$, and $\frac{5}{8}$ to a common denominator.

Ans. $\frac{192}{240}$, $\frac{40}{240}$, $\frac{150}{240}$.

14. Reduce $1\frac{1}{2}$, $\frac{3}{4}$, and $\frac{7}{9}$ to a common denominator.

Ans. $\frac{108}{72}$, $\frac{54}{72}$, $\frac{56}{72}$.

15. Reduce $\frac{7}{10}$, $2\frac{1}{4}$, and $\frac{5}{6}$ to a common denominator.

Ans. $\frac{168}{240}$, $\frac{540}{240}$, $\frac{200}{240}$.

16. Reduce $\frac{5}{12}$, $3\frac{1}{2}$, $\frac{4}{5}$, and $\frac{2}{3}$ to a common denominator.

Ans. $\frac{150}{360}$, $\frac{1260}{360}$, $\frac{288}{360}$, $\frac{240}{360}$.

ADDITION.

95. The denominator of a fraction determines the value of the fractional unit; hence,

I. If two or more fractions have the same denominator, their numerators express fractional units of the same value.

II. If two or more fractions have different denominators, their numerators express fractional units of different values.

And since units of the same value only can be united into one sum, it follows,

III. That fractions can be added only when they have the same fractional unit or common denominator.

1. What is the sum of $\frac{1}{5}$, $\frac{3}{5}$, $\frac{4}{5}$, $\frac{2}{5}$?

ANALYSIS. When fractions have a common denominator, their sum is found by adding their numerators, and placing the sum over the common denominator. Thus, $1+3+4+2=10$, the sum of the numerators; placing this sum over the common denominator 5, we have $\frac{10}{5}=2$, the required sum.

2. What is the sum of $\frac{3}{10}$, $\frac{4}{10}$ and $\frac{7}{10}$?

3. What is the sum of $\frac{2}{7}$, $\frac{5}{7}$, $\frac{1}{7}$ and $\frac{6}{7}$?

4. What is the sum of $\frac{7}{8}$, $\frac{3}{8}$, $\frac{1}{8}$, $\frac{5}{8}$ and $\frac{2}{8}$?

5. A boy paid $\frac{5}{6}$ of a dollar for a pair of gloves, $\frac{2}{6}$ of a dollar for a knife, and $\frac{1}{6}$ of a dollar for a slate; how much did he pay for all?

6. A father distributed some money among his children, as follows: to the first he gave $\frac{5}{12}$ of a dollar, to the second $\frac{3}{12}$, to the third $\frac{7}{12}$, to the fourth $\frac{9}{12}$, and to the fifth $\frac{4}{12}$; how much did he give to all?

7. What is the sum of $\frac{3}{4}$ and $\frac{2}{9}$?

OPERATION.

$\frac{3}{4}+\frac{2}{9}=\frac{27}{36}+\frac{8}{36}=\frac{35}{36}$ *Ans.*

ANALYSIS. As the given fractions have not a common denominator, we reduce them to the same fractional unit, (94) and then add their numerators, $27+8=35$; placing the sum over the common denominator 36, we obtain $\frac{35}{36}$; hence the following

RULE. I. *When the given fractions have the same denominator, add the numerators, and under the sum write the common denominator.*

II. *When they have not the same denominator, reduce them to a common denominator, and then add as before.*

NOTE. If the amount be an improper fraction, reduce it to a whole or a mixed number.

EXAMPLES FOR PRACTICE.

8. What is the sum of $\frac{2}{3}$ and $\frac{4}{5}$? *Ans.* $1\frac{7}{15}$.

9. What is the sum of $\frac{1}{8}$ and $\frac{5}{7}$? *Ans.* $\frac{47}{42}$.

10. What is the sum of $\frac{3}{8}$ and $\frac{2}{5}$? *Ans.* $\frac{31}{40}$.

11. Add $\frac{2}{3}$, $\frac{5}{6}$ and $\frac{3}{8}$ together. *Ans.* $1\frac{7}{8}$.

12. Add $\frac{3}{7}$, $\frac{1}{2}$ and $\frac{2}{9}$ together. *Ans.* $1\frac{19}{126}$.

13. Add $\frac{9}{10}$, $\frac{7}{8}$, $\frac{3}{5}$ and $\frac{1}{4}$ together. *Ans.* $2\frac{5}{8}$.

14. Add $\frac{7}{9}$, $\frac{1}{2}$ and $1\frac{1}{12}$ together. *Ans.* $2\frac{7}{36}$.

15. Add $\frac{4}{5}$, $\frac{1}{7}$, $\frac{2}{4}$ and $\frac{2}{3}$ together. *Ans.* $2\frac{223}{210}$.

16. What is the sum of $\frac{3}{4}$, $\frac{2}{5}$ and $\frac{5}{7}$? *Ans.* $1\frac{121}{140}$.

17. What is the sum of $\frac{2}{5}$, $\frac{5}{6}$ and $\frac{4}{9}$? *Ans.* $1\frac{61}{90}$.

18. What is the sum of $\frac{5}{6}$, $\frac{5}{7}$ and $\frac{5}{8}$? *Ans.* $2\frac{29}{168}$.

To add mixed numbers, *add the fractions and integers separately, and then add their sums.*

NOTE. If the mixed numbers are small, they may be reduced to improper fractions, and then added after the usual method.

19. What is the sum of $14\frac{2}{5}$, $21\frac{1}{2}$ and $9\frac{3}{4}$?

OPERATION.

$14\frac{2}{5}=14\frac{8}{20}$
$21\frac{1}{2}=21\frac{10}{20}$
$9\frac{3}{4}=9\frac{15}{20}$
$45\frac{13}{20}$ *Ans.*

ANALYSIS. By reducing the fractions to a common denominator, and adding them, we obtain $\frac{33}{20}$ or $1\frac{13}{20}$, which added to the sum of the integral numbers, gives $45\frac{13}{20}$, the *Ans.*

20. What is the sum of $3\frac{1}{4}$, $12\frac{5}{6}$ and $25\frac{2}{3}$? *Ans.* $41\frac{3}{4}$.

21. What is the sum of $\frac{7}{8}$, $15\frac{1}{2}$, $42\frac{1}{3}$ and 50?

22. What is the sum of $30\frac{2}{8}$, $1\frac{1}{5}$, $16\frac{7}{10}$ and $\frac{19}{20}$?

23. Bought 3 pieces of cloth containing $45\frac{1}{2}$, $38\frac{1}{3}$, and $35\frac{1}{4}$ yards; how many yards in the 3 pieces?

Ans. $119\frac{1}{12}$ yards.

24. Three men bought a horse. A paid $31\frac{3}{8}$ dollars, B paid $43\frac{5}{12}$ dollars, and C paid $47\frac{5}{6}$ dollars; what was the cost of the horse? *Ans.* $122\frac{5}{8}$ dollars.

25. If it take $5\frac{1}{2}$ yards of cloth for an overcoat, $4\frac{1}{8}$ yards for a dress coat, $2\frac{1}{3}$ yards for a pair of pantaloons, and $\frac{7}{8}$ of a yard for a vest, how many yards of cloth will it take for the whole suit? *Ans.* $12\frac{5}{8}$ yards.

SUBTRACTION.

96. The process of subtracting one fraction from another is based upon the following principles:

I. One number can be subtracted from another only when the two numbers have the same unit value. Hence,

II. In subtraction of fractions, the minuend and subtrahend must have a common denominator.

1. From $\frac{9}{12}$ subtract $\frac{5}{12}$.

ANALYSIS. Since the fractions have a common denominator, the difference is obtained, by subtracting the less numerator 5, from the greater 9, and writing the result over the common denominator 12; we thus obtain $\frac{4}{12}$, the required difference.

2. From $\frac{7}{8}$ subtract $\frac{3}{8}$.

3. From $\frac{11}{15}$ subtract $\frac{9}{15}$.

4. Subtract $\frac{17}{24}$ from $\frac{21}{24}$.

5. James had $\frac{7}{8}$ of a bushel of walnuts, and sold $\frac{4}{8}$ of them; how many had he left?

6. Harvey had $\frac{15}{16}$ of a dollar, and gave $\frac{5}{16}$ of a dollar to a beggar; how much had he left?

7. Subtract $\frac{3}{7}$ from $\frac{2}{3}$.

OPERATION. $\frac{2}{3}-\frac{3}{7}=\frac{14}{21}-\frac{9}{21}=\frac{5}{21}$ *Ans.*

ANALYSIS. As the given fractions have not a common denominator, we first reduce them to the same fractional unit, (94) and then subtract the less numerator 9, from the greater 14, and write the result over the common denominator 21. We thus obtain $\frac{5}{21}$ the required difference. Hence the following

RULE. I. *When the fractions have the same denominator, subtract the less numerator from the greater, and place the result over the common denominator.*

II. *When they have not a common denominator, reduce them to a common denominator before subtracting.*

EXAMPLES FOR PRACTICE.

8. From $\frac{7}{8}$ take $\frac{3}{4}$. *Ans.* $\frac{1}{8}$.
9. From $\frac{2}{3}$ take $\frac{4}{9}$. *Ans.* $\frac{2}{9}$.
10. From $\frac{5}{7}$ take $\frac{2}{5}$. *Ans.* $\frac{11}{35}$.
11. From $\frac{9}{10}$ take $\frac{1}{6}$. *Ans.* $\frac{11}{15}$.
12. Subtract $\frac{5}{9}$ from $\frac{3}{4}$. *Ans.* $\frac{7}{36}$.
13. Subtract $\frac{9}{20}$ from $\frac{4}{5}$. *Ans.* $\frac{7}{20}$.
14. Subtract $\frac{4}{5}$ from $1\frac{1}{13}$. *Ans.* $\frac{29}{65}$.
15. Subtract $\frac{7}{18}$ from $1\frac{1}{12}$. *Ans.* $\frac{19}{36}$.
16. Subtract $\frac{11}{24}$ from $\frac{2}{3}$. *Ans.* $\frac{5}{24}$.
17. Subtract $\frac{21}{30}$ from $\frac{7}{8}$. *Ans.* $\frac{7}{40}$.
18. From $9\frac{1}{3}$ take $2\frac{3}{4}$.

OPERATION.

$9\frac{1}{3}=9\frac{4}{12}$
$2\frac{3}{4}=2\frac{9}{12}$
$6\frac{7}{12}$ *Ans.*

ANALYSIS. We first reduce the fractional parts, $\frac{1}{3}$ and $\frac{3}{4}$, to a common denominator 12. Since we cannot take $\frac{9}{12}$ from $\frac{4}{12}$, we add $1=\frac{12}{12}$ to $\frac{4}{12}$, which makes $\frac{16}{12}$, and $\frac{9}{12}$ from $\frac{16}{12}$ leaves $\frac{7}{12}$. We now add 1 to the 2 in the subtrahend, and say, 3 from 9 leaves 6. We thus obtain $6\frac{7}{12}$, the difference required.

Hence, to subtract mixed numbers, we may *reduce the fractional parts to a common denominator, and then subtract the fractional and integral parts separately.*

19. From $24\frac{3}{4}$ take $17\frac{1}{2}$. *Ans.* $7\frac{1}{4}$.
20. From $147\frac{2}{3}$ take $49\frac{1}{4}$. *Ans.* $98\frac{5}{12}$.
21. From $75\frac{1}{3}$ take $40\frac{3}{5}$. *Ans.* $34\frac{11}{15}$.
22. From $63\frac{3}{10}$ take $22\frac{2}{6}$. *Ans.* $40\frac{4}{5}$.
23. Bought flour at $6\frac{1}{2}$ dollars a barrel, and sold it at $7\frac{7}{8}$ dollars a barrel; what was the gain per barrel?

Ans. $\frac{9}{10}$ of a dollar.

24. From a cask of wine containing $38\frac{5}{8}$ gallons, $15\frac{5}{6}$ gallons were drawn; how many gallons remained?

Ans. $22\frac{19}{24}$ gallons

MULTIPLICATION.

CASE I.

97. To multiply a fraction by an integer.

1. If 1 pound of sugar cost $\frac{1}{8}$ of a dollar, how much will 3 pounds cost?

ANALYSIS. If 1 pound cost $\frac{1}{8}$ of a dollar, 3 pounds, which are 3 times 1 pound, will cost 3 times $\frac{1}{8}$ or $\frac{3}{8}$ of a dollar. Therefore, 3 pounds of sugar, at $\frac{1}{8}$ of a dollar a pound, will cost $\frac{3}{8}$ of a dollar.

2. If 1 horse eat $\frac{2}{9}$ of a ton of hay in 1 month, how much will 4 horses eat in the same time?

3. At $\frac{3}{5}$ of a dollar a bushel, what will be the cost of 2 bushels of pears? of 3 bushels? of 5 bushels?

4. How many are 3 times $\frac{2}{3}$? 5 times $\frac{3}{4}$? 4 times $\frac{1}{6}$? 6 times $\frac{2}{7}$? 9 times $\frac{1}{10}$? 8 times $\frac{3}{5}$?

5. If one yard of cloth cost $\frac{5}{6}$ of a dollar, how much will 3 yards cost?

FIRST OPERATION.

$\frac{5}{6} \times 3 = \frac{15}{6} = 2\frac{1}{2}$.

SECOND OPERATION.

$\frac{5}{6} \times 3 = \frac{5}{2} = 2\frac{1}{2}$.

ANALYSIS. In the first operation we multiply the fraction by 3, by multiplying its numerator by 3, obtaining $\frac{15}{6} = 2\frac{1}{2}$. In this case the *value* of the fractional unit remains the same, but we multiply the *number taken*, 3 times. In the second operation, we multiply the fraction by 3, by dividing its denominator by 3, obtaining $\frac{5}{2} = 2\frac{1}{2}$. In this case, the *value* of the fractional unit is multiplied, 3 times, but the *number taken*, is the same. Hence,

Multiplying a fraction consists in multiplying its numerator, or dividing its denominator.

NOTE. Always divide the denominator when it is exactly divisible by the multiplier.

EXAMPLES FOR PRACTICE.

6. Multiply $\frac{6}{7}$ by 5. *Ans.* $4\frac{2}{7}$.
7. Multiply $\frac{7}{9}$ by 4. *Ans.* $3\frac{1}{9}$.
8. Multiply $\frac{9}{10}$ by 6. *Ans.* $5\frac{2}{5}$.
9. Multiply $\frac{12}{27}$ by 9. *Ans.* 4.
10. Multiply $\frac{9}{24}$ by 3. *Ans.* $1\frac{1}{8}$.
11. Multiply $\frac{30}{42}$ by 14. *Ans.* 10.
12. Multiply $4\frac{1}{3}$ by 5.

OPERATION.

$4\frac{1}{3}$
5
—
$1\frac{2}{3}$
20
—
$21\frac{2}{3}$

Or,

$4\frac{1}{3}=\frac{13}{3}$

$\frac{13}{3}\times 5=\frac{65}{3}=21\frac{2}{3}$

ANALYSIS. In multiplying a mixed number, we first multiply the fractional part, then the integer, and then add the two products. Thus, $5\times\frac{1}{3}=\frac{5}{3}=1\frac{2}{3}$; and $5\times 4=20$, which added to $1\frac{2}{3}$, gives $21\frac{2}{3}$, the required result. Or, we may reduce the mixed number to an improper fraction, and then multiply it.

13. Multiply $6\frac{3}{4}$ by 8. *Ans.* 54.
14. Multiply $9\frac{4}{5}$ by 7. *Ans.* $68\frac{3}{5}$.
15. If a man earn $1\frac{7}{8}$ in 1 day, how much will he earn in 10 days? *Ans.* $18\frac{3}{4}$ dollars.
16. What will 14 yards of cloth cost, at $\frac{5}{7}$ of a dollar a yard? *Ans.* 10 dollars.
17. At $3\frac{1}{4}$ dollars a cord, what will be the cost of 20 cords of wood? *Ans.* 65 dollars.
18. If one man can mow $1\frac{9}{10}$ acres of grass in a day, how many acres can 5 men mow? *Ans.* $9\frac{1}{2}$ acres.
19. What will 9 dozen eggs cost, at $14\frac{1}{2}$ cents a dozen? *Ans.* $130\frac{1}{2}$ cents.
20. At $6\frac{4}{5}$ dollars a barrel, what will 30 barrels of flour cost? *Ans.* 204 dollars.

CASE II.

98. To multiply an integer by a fraction.

1. At 9 dollars a barrel, what will $\frac{2}{3}$ of a barrel of flour cost?

ANALYSIS. Since 1 barrel of flour cost 9 dollars, $\frac{2}{3}$ of a barrel will cost 2 times $\frac{1}{3}$ of 9 dollars. $\frac{1}{3}$ of 9 dollars is 3 dollars, and $\frac{2}{3}$ of 9 dollars is 2 times 3 dollars, or 6 dollars. Therefore $\frac{2}{3}$ of a barrel will cost 6 dollars.

2. If a yard of cloth be worth 8 dollars, what is $\frac{3}{4}$ of a yard worth?

3. If an acre of land produce 25 bushels of wheat, how much will $\frac{1}{5}$ of an acre produce? $\frac{3}{5}$ of an acre? $\frac{4}{5}$ of an acre?

4. If a man earn 20 dollars in a month, how much can he earn in $\frac{1}{2}$ of a month? in $\frac{3}{5}$? in $\frac{7}{10}$? in $\frac{3}{4}$?

5. If a ton of hay cost 12 dollars, how much will $\frac{1}{4}$ of a ton cost? $\frac{3}{4}$ of a ton? $\frac{2}{3}$ of a ton? $\frac{5}{6}$ of a ton?

6. At 60 dollars an acre, what will $\frac{4}{5}$ of an acre of land cost?

FIRST OPERATION.

5)60 price of 1 acre.
12 cost of $\frac{1}{5}$ of an acre.
4
48 cost of $\frac{4}{5}$ of an acre.

SECOND OPERATION.

60 price of 1 acre.
4
5)240 cost of 4 acres.
48 cost of $\frac{4}{5}$ of an acre.

ANALYSIS. 4 fifths of an acre will cost 4 times as much as 1 fifth of an acre, or 4 times $\frac{1}{5}$ of 60 dollars. $\frac{1}{5}$ of 60 dollars is 12 dollars, and $\frac{4}{5}$ is 4 times 12, or 48 dollars, the cost of $\frac{4}{5}$ of an acre. In the second operation, we multiply the price of 1 acre by 4, and obtain 240 dollars, the cost of 4 acres; but as $\frac{4}{5}$ of 1 acre is the same as $\frac{1}{5}$ of 4 acres, we divide 240 dollars, the cost of 4 acres, by 5, and obtain 48 dollars, the cost of of $\frac{4}{5}$ of acre. Hence,

RULE. *Multiplying an integer by a fraction, consists in multiplying by the numerator, and dividing the product by the denominator.*

7. Multiply 45 by $\frac{3}{4}$. *Ans.* $33\frac{3}{4}$.
8. Multiply 68 by $\frac{4}{5}$. *Ans.* $54\frac{2}{5}$.
9. Multiply 105 by $\frac{7}{15}$. *Ans.* 49.
10. Multiply 480 by $\frac{5}{8}$. *Ans.* 300.
11. At 16 dollars a ton, what will be the cost of $\frac{3}{4}$ of a ton of hay? *Ans.* 12 dollars.
12. If a village lot is worth 340 dollars, what is $\frac{3}{4}$ of it worth? *Ans.* 255 dollars.
13. If a hogshead of sugar is worth 75 dollars, what is $\frac{11}{12}$ of it worth? *Ans.* $68\frac{3}{4}$ dollars.
14. If an acre of land produce 114 bushels of oats, how many bushels will $\frac{9}{16}$ of an acre produce?
Ans. $64\frac{1}{8}$ bushels.
15. If a man travel 47 miles in a day, how far does he travel in $\frac{5}{8}$ of a day? *Ans.* $29\frac{3}{8}$ miles.

CASE III.

99. To multiply a fraction by a fraction.

1. If a bushel of apples is worth $\frac{1}{4}$ of a dollar, what is $\frac{1}{2}$ of a bushel worth?

ANALYSIS. Since 1 bushel is worth $\frac{1}{4}$ of a dollar, $\frac{1}{2}$ of a bushel is worth $\frac{1}{2}$ times $\frac{1}{4}$ of a dollar; $\frac{1}{4}$ equals $\frac{2}{8}$, and a $\frac{1}{2}$ of $\frac{2}{8}$ is $\frac{1}{8}$. Therefore $\frac{1}{2}$ of a bushel is worth $\frac{1}{8}$ of a dollar.

2. If a yard of cloth cost $\frac{1}{2}$ a dollar, how much will $\frac{1}{3}$ of a yard cost?

3. When oats are worth $\frac{1}{3}$ of a dollar a bushel, what is $\frac{1}{4}$ of a bushel worth.

4. If a man own $\frac{1}{2}$ of a vessel, and he sells $\frac{1}{5}$ of his share what part of the vessel does he sell?

5. At $\frac{2}{3}$ of a dollar a bushel, what will $\frac{3}{4}$ of a bushel of corn cost?

OPERATION.

$\frac{2}{3}\times\frac{3}{4}=\frac{6}{12}=\frac{1}{2}$ *Ans.*

ANALYSIS. Since 1 bushel cost $\frac{2}{3}$ of a dollar, $\frac{3}{4}$ of a bushel will cost $\frac{3}{4}$ times $\frac{2}{3}$ of a dollar. By multiplying the numerators 2 and 3 together, we obtain the numerator 6 of the product; and by multiplying the denominators 3 and 4 together, we obtain the denominator 12 of the product, and thus we have $\frac{6}{12}=\frac{1}{2}$ for the required product. Hence we have the following

RULE. *Multiply together the numerators for a new numerator, and the denominators for a new denominator, and reduce the result to its lowest terms.*

EXAMPLES FOR PRACTICE.

6. Multiply $\frac{1}{2}$ by $\frac{3}{5}$. *Ans.* $\frac{3}{10}$.
7. Multiply $\frac{3}{4}$ by $\frac{3}{7}$. *Ans.* $\frac{9}{28}$.
8. Multiply $\frac{4}{5}$ by $\frac{6}{7}$. *Ans.* $\frac{24}{35}$.
9. Multiply $\frac{3}{8}$ by $\frac{5}{9}$. *Ans.* $\frac{5}{24}$.
10. Multiply $\frac{7}{12}$ by $\frac{5}{7}$. *Ans.* $\frac{5}{12}$.
11. What is the product of $\frac{2}{5}$, $\frac{1}{3}$ and $\frac{3}{4}$? *Ans.* $\frac{1}{10}$.
12. What is the product of $\frac{5}{8}$, $\frac{1}{2}$ and $\frac{2}{9}$? *Ans.* $\frac{5}{72}$.
13. What is the product of $\frac{7}{9}$, $\frac{2}{7}$ and $\frac{5}{10}$? *Ans.* $\frac{1}{9}$.
14. What is the product of $\frac{1}{2}\frac{3}{4}$ and $\frac{1}{2}\frac{6}{6}$? *Ans.* $\frac{1}{3}$.
15. What is the product of $\frac{5}{6}$, $1\frac{1}{2}$, 5 and $\frac{4}{5}$?

OPERATION.

$\frac{5}{6}\times1\frac{1}{2}\times5\times\frac{4}{5}=$

$\frac{5}{6}\times\frac{3}{2}\times\frac{5}{1}\times\frac{4}{5}=\frac{10}{2}=5$ *Ans.*

When *integers* or *mixed numbers* occur among the given factors, they may be reduced to improper fractions before multiplying; and an *integer* may be reduced to the form of a fraction by writing 1 for its denominator; thus $5=\frac{5}{1}$.

16. What is the product of $\frac{3}{7}$, $\frac{2}{9}$ and $2\frac{2}{3}$? *Ans.* $\frac{16}{63}$.
17. What is the product of 3, $\frac{9}{10}$ and $\frac{5}{6}$? *Ans.* $2\frac{1}{4}$.

18. What is the product of $\frac{3}{7}$, $\frac{5}{11}$ and $\frac{22}{25}$? *Ans.* $\frac{6}{35}$.

19. Find the value of $\frac{2}{3}$ of $\frac{5}{8}$ multiplied by $\frac{3}{5}$ of $\frac{9}{11}$.

OPERATION.

$$\frac{2}{3}\times\frac{5}{8}\times\frac{3}{5}\times\frac{9}{11}=\frac{9}{44}\ \textit{Ans.}$$

NOTES. 1. Fractions with the word *of* between them are sometimes called *compound fractions*. The word *of* is simply an equivalent for the sign of multiplication, and signifies that the numbers between which it is placed are to be multiplied together.

2. When the same factors occur in both numerator and denominator of fractions to be multiplied together, they may be omitted and the remaining factors only used; thus, 5 and 3 being found in both the numerators and denominators of the above example may be omitted in multiplying.

20. Multiply $\frac{3}{4}$ of $\frac{6}{7}$ by $\frac{4}{9}$ of $\frac{1}{6}$. *Ans.* $\frac{1}{21}$.

21. Multiply $\frac{4}{5}$ of 3 by $\frac{7}{8}$ of $2\frac{1}{2}$. *Ans.* $5\frac{1}{4}$.

22. What is the product of $\frac{9}{10}$, $\frac{1}{2}$ of $\frac{5}{9}$ and $1\frac{1}{3}$?

Ans. $\frac{1}{3}$.

23. What is the product of $\frac{6}{7}$ of $\frac{7}{11}$ by $5\frac{1}{2}$? *Ans.* 3.

24. What is the value of $\frac{3}{5}$ times $\frac{1}{6}$ of $\frac{2}{3}$ of 10?

Ans. $\frac{2}{3}$

25. What is the value of $\frac{5}{12}$ of $\frac{4}{5}$ times $\frac{1}{4}$ of $3\frac{3}{7}$?

Ans. $\frac{2}{7}$.

26. At $\frac{8}{9}$ of a dollar a bushel, what will $\frac{3}{8}$ of a bushel of corn cost? *Ans.* $\frac{1}{3}$ of a dollar.

27. When peaches are worth $\frac{9}{10}$ of a dollar a bushel, what is $\frac{5}{9}$ of a bushel worth? *Ans.* $\frac{1}{2}$ dollar.

28. Jane having $\frac{4}{5}$ of a yard of silk gave $\frac{3}{4}$ of it to her sister; what part of a yard did she give her sister?

Ans. $\frac{3}{5}$ of a yard.

29. When pears are worth $\frac{7}{8}$ of a dollar a basket, what is $\frac{4}{7}$ of $\frac{3}{4}$ of a basket worth? *Ans.* $\frac{3}{8}$ of a dollar.

30. A man owning $\frac{13}{20}$ of a ship, sold $\frac{5}{8}$ of his share; what part of the whole ship did he sell? *Ans.* $\frac{13}{32}$.

31. A grocer having $\frac{15}{21}$ of a hogshead of molasses sold $\frac{7}{10}$ of it; what part of a hogshead remained?

32. At $\frac{5}{8}$ of a dollar a yard, what will be the cost of $\frac{4}{5}$ of 3 yards of cloth? *Ans.* $1\frac{1}{2}$ dollars.

DIVISION.

CASE I.

100. To divide a fraction by an integer.

1. If 3 pounds of raisins cost $\frac{1}{2}$ of a dollar, how much will 1 pound cost?

ANALYSIS. If 3 pounds cost $\frac{1}{2}$ of a dollar, 1 pound which is $\frac{1}{3}$ of 3 pounds, will cost $\frac{1}{3}$ of $\frac{1}{2}$, or $\frac{1}{6}$ of a dollar. Therefore, 1 pound will cost $\frac{1}{6}$ of a dollar.

2. If 4 pounds of coffee cost $\frac{1}{3}$ of a dollar, how much will 1 pound cost?

3. If 5 marbles cost $\frac{1}{8}$ of a dollar, how much will 1 marble cost?

4. If $\frac{1}{5}$ of a barrel of flour be equally divided among 6 persons, what part of a barrel will each have?

5. If $\frac{1}{7}$ of a box of tea be equally distributed among 8 persons, what part of a box will each have?

6. Paid $\frac{8}{9}$ of a dollar for 4 pounds of butter; what was the cost per pound?

FIRST OPERATION.
$\frac{8}{9} \div 4 = \frac{2}{9}$ *Ans.*

SECOND OPERATION.
$\frac{8}{9} \div 4 = \frac{8}{36} = \frac{2}{9}$ *Ans.*

ANALYSIS. In the first operation we divide the fraction by 4, by dividing its numerator by 4, obtaining $\frac{2}{9}$. In this case the *value* of the fractional unit is unchanged, but we diminish the *number taken*, 4 times. In the second operation we divide the fraction by 4, by multiplying the denominator by 4, obtaining $\frac{8}{36} = \frac{2}{9}$. In this case the *value* of the fractional unit is diminished 4 times, but the *number taken* is the same. Hence,

Dividing a fraction consists in dividing its numerator, or multiplying its denominator.

NOTE. We divide the numerator when it is exactly divisible by the divisor; otherwise we multiply the denominator

EXAMPLES FOR PRACTICE.

7. Divide $\frac{6}{7}$ by 3. *Ans.* $\frac{2}{7}$.
8. Divide $\frac{4}{9}$ by 4. *Ans.* $\frac{1}{9}$.
9. Divide $\frac{10}{13}$ by 5. *Ans.* $\frac{2}{13}$.
10. Divide $\frac{15}{16}$ by 5. *Ans.* $\frac{3}{16}$.
11. Divide $\frac{1}{7}$ by 9. *Ans.* $\frac{1}{63}$.
12. Divide $\frac{42}{47}$ by 21. *Ans.* $\frac{2}{47}$.
13. Divide $\frac{3}{4}$ of $\frac{8}{9}$ by 12. *Ans.* $\frac{1}{18}$.
14. Divide $\frac{4}{5}$ of $\frac{2}{3}$ by 6. *Ans.* $\frac{4}{45}$.
15. Divide $4\frac{1}{5}$ by 7.

OPERATION.

$4\frac{1}{5}=\frac{21}{5}$
$\frac{21}{5}\div 7=\frac{3}{5}$ *Ans.*

NOTE. We reduce the mixed number to an improper fraction and then divide as before.

16. Divide $3\frac{4}{5}$ by 4. *Ans.* $\frac{19}{20}$.
17. Divide $6\frac{3}{8}$ by 9. *Ans.* $\frac{17}{24}$.
18. Divide $\frac{1}{2}$ of $2\frac{1}{4}$ by 3. *Ans.* $\frac{3}{8}$.
19. Divide $8\frac{7}{10}$ by 12. *Ans.* $\frac{29}{40}$.
20. Divide $13\frac{3}{4}$ by 10. *Ans.* $1\frac{3}{8}$.
21. Divide $\frac{5}{6}$ of 8 by 20. *Ans.* $\frac{1}{3}$.

22. If 6 persons agree to share equally $\frac{3}{4}$ of a bushel of grapes, what part of a bushel will each have? *Ans.* $\frac{1}{8}$.

23. If 5 yards of sheeting cost $\frac{9}{10}$ of a dollar, what will 1 yard cost? *Ans.* $\frac{9}{50}$ of a dollar.

24. If 8 bushels of apples cost $5\frac{3}{9}$ dollars, what will 1 bushel cost? *Ans.* $\frac{2}{3}$ of a dollar.

25. If $\frac{1}{2}$ of 10 pounds of butter cost $1\frac{1}{4}$ dollars, what will 1 pound cost? *Ans.* $\frac{1}{4}$ of a dollar.

26. A man distributed $\frac{16}{20}$ of a dollar equally among 6 beggars; what part of a dollar did he give to each?

27. If $\frac{2}{3}$ of 9 cords of wood cost $12\frac{3}{8}$ dollars, what will 1 cord cost?

CASE II.

101. To divide an integer by a fraction.

1. At $\frac{1}{3}$ of a dollar a yard, how many yards of ribbon can be bought for 2 dollars?

ANALYSIS. As many yards as $\frac{1}{3}$ of a dollar, the price of 1 yard, is contained times in 2 dollars. Since in 1 dollar there are 3 thirds of a dollar, in two dollars, there are 2 times 3 thirds, or 6 thirds; and 1 third is contained in 6 thirds, 6 times. Therefore 6 yards of ribbon can be bought for 2 dollars.

2. When potatoes are $\frac{3}{4}$ of a dollar a bushel, how many bushels can be bought for 2 dollars? for 4 dollars? for 6 dollars?

3. If a man spend $\frac{1}{3}$ of a dollar a day for cigars, how long will it take him to spend 3 dollars? 5 dollars? 6 dollars?

4. At $\frac{4}{5}$ of a dollar a bushel, how many bushels of corn can be bought for 16 dollars?

FIRST OPERATION.

16
5
—
4)80
—
20 bushels.

SECOND OPERATION.

$4)\frac{16}{}$
$\frac{4}{}$
5
—
20 bushels.

ANALYSIS. As many bushels as $\frac{4}{5}$ of a dollar, the price of 1 bushel, is contained times in 16 dollars. But we cannot divide *integers* by *fifths*, because they are not of the same denomination. Reducing 16 dollars to *fifths* by multiplying by 5, we have 80 fifths, and 4 *fifths* is contained in 80 *fifths*, 20 times, the required number of bushels. In the second operation, we divide the integer by the numerator of the fraction, and multiply the quotient by the denominator, which produces the same result as in the first operation. Hence

Dividing by a fraction consists in multiplying by the denominator, and dividing the product by the numerator of the divisor.

EXAMPLES FOR PRACTICE.

5. Divide 18 by $\frac{2}{3}$. *Ans.* 27.
6. Divide 14 by $\frac{2}{7}$. *Ans.* 49.
7. Divide 11 by $\frac{5}{9}$. *Ans.* $19\frac{4}{5}$.
8. Divide 75 by $\frac{9}{10}$. *Ans.* $83\frac{1}{3}$.
9. Divide 120 by $\frac{6}{11}$. *Ans.* 220.
10. Divide 96 by $\frac{12}{7}$. *Ans.* 136.
11. Divide 226 by $\frac{9}{25}$. *Ans.* $627\frac{7}{9}$.
12. Divide 28 by $4\frac{2}{3}$.

OPERATION.

$28 \times 3 = 84$
$84 \div 14 = 6$ *Ans.*

ANALYSIS. We reduce the mixed number to an improper fraction, and then divide the integer in the same manner as by a proper fraction.

13. Divide 16 by $2\frac{1}{4}$. *Ans.* $7\frac{1}{9}$.
14. Divide 42 by $3\frac{1}{2}$. *Ans.* 12.
15. Divide 112 by $6\frac{2}{5}$. *Ans.* $17\frac{1}{2}$.
16. Divide 180 by $7\frac{1}{8}$. *Ans.* $25\frac{5}{19}$.
17. Divide 425 by $\frac{5}{7}$. *Ans.* 595.
18. Divide 318 by $\frac{6}{23}$. *Ans.* 1219.

19. When potatoes are $\frac{7}{9}$ of a dollar a bushel, how many bushels can be bought for 10 dollars? *Ans.* $12\frac{6}{7}$ bush.

20. Divide 9 bushels of corn among some persons, giving them $\frac{3}{16}$ of a bushel each; how many persons will receive a share? *Ans.* 48.

21. At $2\frac{4}{5}$ dollars a cord, how many cords of wood can be bought for 27 dollars? *Ans.* $9\frac{9}{14}$ cords.

22. If a horse eat $\frac{5}{9}$ of a bushel of oats in a day, in how many days will he eat 20 bushels? *Ans.* 36 days.

23. If a man walk $2\frac{9}{10}$ miles an hour, how many hours will he require to walk 48 miles? *Ans.* $16\frac{16}{29}$ hours.

24. At $\frac{1}{16}$ of a dollar a pound, how many pounds of rice can be bought for 3 dollars? *Ans.* 48 pounds.

CASE III.

102. To divide a fraction by a fraction.

1. At $\frac{2}{5}$ of a dollar a pound, how many pounds of tea can be bought for $\frac{4}{5}$ of a dollar?

ANALYSIS. As many pounds as $\frac{2}{5}$ of a dollar, the price of 1 pound, is contained times in $\frac{4}{5}$ of a dollar; 2 *fifths* are contained in 4 *fifths*, 2 times. Therefore 2 pounds can be bought for $\frac{4}{5}$ of a dollar.

Hence we see, that when fractions have a common denominator, division may be performed by dividing the numerator of the dividend by the numerator of the divisor.

2. How many pine-apples at $\frac{3}{10}$ of a dollar each, can be bought for $\frac{6}{10}$ of a dollar? for $\frac{9}{10}$? for $\frac{12}{10}$?

3. If a horse eat $\frac{2}{7}$ of a bushel of oats in 1 day, in how many days will he eat $\frac{4}{7}$ of a bushel? $\frac{6}{7}$? $\frac{10}{7}$? $\frac{14}{7}$?

4. At $\frac{1}{3}$ of a dollar a bushel, how many bushels of apples can be bought for $\frac{2}{3}$ of a dollar? for $\frac{4}{3}$? for $\frac{6}{3}$?

5. At $\frac{2}{5}$ of a dollar a pound, how many pounds of tea can be bought for $\frac{3}{4}$ of a dollar?

FIRST OPERATION.

$\frac{2}{5}=\frac{8}{20}$; $\frac{3}{4}=\frac{15}{20}$.

$\frac{15}{20}\div\frac{8}{20}=1\frac{7}{8}$ *Ans.*

SECOND OPERATION

$\frac{3}{4}\div\frac{2}{5}=\frac{3}{4}\times\frac{5}{2}=\frac{15}{8}=1\frac{7}{8}$ *Ans.*

ANALYSIS. As many pounds as $\frac{2}{5}$ of a dollar, the price of 1 pound, is contained times in $\frac{3}{4}$ of a dollar. $\frac{2}{5}$ equal $\frac{8}{20}$, $\frac{3}{4}$ equal $\frac{15}{20}$, and 8 *twentieths* are contained in 15 *twentieths* $1\frac{7}{8}$ times. Or, as in the second operation, we have multiplied the dividend $\frac{3}{4}$, by the denominator 5, of the divisor, and divided the result by the numerator 2, of the divisor. Hence, by inverting the terms of the divisor the two fractions will stand in such relation to each other, that we can multiply together the two upper numbers for the numerator of the quotient, and the two lower numbers for the denominator, as shown in the second operation. Hence

RULE. I. *Reduce integers and mixed numbers to improper fractions.*

II. *Invert the terms of the divisor, and proceed as in multiplication.*

EXAMPLES FOR PRACTICE.

6. Divide $\frac{9}{10}$ by $\frac{3}{10}$. *Ans.* 3.
7. Divide $\frac{1}{2}$ by $\frac{1}{4}$. *Ans.* 2.
8. Divide $\frac{5}{8}$ by $\frac{2}{3}$. *Ans.* $1\frac{5}{16}$.
9. Divide $\frac{7}{8}$ by $\frac{2}{5}$. *Ans.* $2\frac{3}{16}$.
10. How many times is $\frac{7}{19}$ contained in $\frac{17}{19}$? *Ans.* $2\frac{3}{7}$.
11. How many times is $\frac{6}{7}$ contained in $\frac{3}{5}$? *Ans.* $\frac{7}{10}$.
12. How many times is $\frac{1}{2}$ contained in $\frac{15}{16}$? *Ans.* $1\frac{7}{8}$.
13. Divide $\frac{1}{2}$ of $\frac{3}{4}$ by $\frac{7}{8}$. *Ans.* $\frac{3}{7}$.
14. Divide $\frac{4}{5}$ of $\frac{5}{8}$ by $\frac{7}{20}$. *Ans.* $1\frac{3}{7}$.
15. Divide $\frac{11}{15}$ by $\frac{1}{7}$ of $\frac{2}{3}$. *Ans.* $7\frac{7}{10}$.
16. Divide $\frac{3}{4}$ of $\frac{1}{2}$ by $\frac{2}{5}$ of $\frac{7}{8}$. *Ans.* $1\frac{1}{14}$.

17. At $\frac{1}{7}$ of a dollar a pound, how many pounds of sugar can be bought for $\frac{4}{5}$ of a dollar? *Ans.* $5\frac{3}{5}$ pounds.

18. At $\frac{7}{10}$ of a dollar a pint, how much wine can be bought for $\frac{1}{2}$ of a dollar? *Ans.* $\frac{5}{7}$ of a pint.

19. At $\frac{4}{5}$ of $\frac{1}{3}$ of a dollar a yard, how many yards of ribbon can be bought for $\frac{7}{10}$ of a dollar? *Ans.* $2\frac{5}{8}$ yards.

20. At $\frac{7}{9}$ of a dollar a yard, how many yards of silk can be bought for $\frac{5}{3}$ of a dollar? *Ans.* $2\frac{1}{7}$ yards.

21. A man owning $\frac{5}{8}$ of a copper mine, divided his share equally among his sons, giving them $\frac{5}{16}$ each; how many sons had he? *Ans.* 2.

22. If $\frac{6}{7}$ of a bushel of pears cost $\frac{3}{5}$ of a dollar, how much will 1 bushel cost? *Ans.* $\frac{7}{10}$ of a dollar.

23. How much corn at $\frac{8}{9}$ of a dollar a bushel, can be bought for $\frac{2}{3}$ of a dollar. *Ans.* $\frac{3}{4}$ of a bushel.

PROMISCUOUS EXAMPLES.

1. In $25\frac{9}{16}$ pounds how many 16ths of a pound?
2. Reduce $4\frac{21}{36}$ to a mixed number. *Ans.* $11\frac{25}{36}$.
3. Reduce $\frac{156}{208}$ to its lowest terms. *Ans.* $\frac{3}{4}$.
4. In $7\frac{859}{8}$ of a day how many days?
5. Change 42 pounds to *sevenths* of a pound.
6. Reduce $21\frac{4}{5}$ to an improper fraction. *Ans.* $\frac{109}{5}$.
7. Reduce $126\frac{2}{3}$ to *thirds*. *Ans.* $\frac{380}{3}$.
8. Reduce $\frac{288}{360}$ to its lowest terms. *Ans.* $\frac{4}{5}$.
9. Reduce $\frac{1}{2}$ and $\frac{4}{5}$ to a common denominator.
10. Reduce 36 to a fraction whose denominator is 12.
11. What is the sum of $\frac{3}{4}$, $\frac{5}{8}$ and $\frac{1}{2}$? *Ans.* $1\frac{7}{8}$.
12. Add together $\frac{9}{10}$, $\frac{1}{2}$ and $3\frac{1}{5}$. *Ans.* $4\frac{3}{5}$.
13. What is the difference between $\frac{7}{8}$ and $\frac{2}{3}$?
14. Reduce $\frac{9}{16}$, $\frac{7}{8}$ and $\frac{3}{4}$ to a common denominator.
15. Sold $9\frac{3}{8}$ cords of wood to one man, and $12\frac{9}{16}$ to another; how much did I sell to both?
16. Paid $87\frac{9}{10}$ dollars for a horse, and $62\frac{1}{2}$ dollars for a wagon; how much more was paid for the horse than the wagon? *Ans.* $25\frac{2}{5}$ dollars.
17. A farmer having $234\frac{15}{16}$ acres of land, sells at one time $42\frac{3}{4}$ acres, at another time $61\frac{3}{8}$, and at another $70\frac{1}{2}$ acres; how many acres has he left? *Ans.* $60\frac{5}{16}$ acres.
18. A speculator bought 120 bushels of wheat, for $136\frac{2}{5}$ dollars, and sold it for $197\frac{5}{8}$ dollars; how much did he gain?
19. Bought 12 pounds of coffee at $\frac{1}{8}$ of a dollar a pound, and 9 pounds of tea at $\frac{4}{5}$ of a dollar a pound; what was the cost of the whole? *Ans.* $8\frac{7}{10}$ dollars.
20. Bought 10 bushels of wheat, at $1\frac{1}{3}$ dollars a bushel, and 14 bushels of corn, at $\frac{5}{7}$ of a dollar a bushel; which cost the more, and how much?

Ans. the corn, $3\frac{1}{3}$ dollars.

21. Paid 12 dollars for some cloth, at the rate of $\frac{3}{4}$ of a dollar a yard; how many yards was purchased?

22. If 8 oranges cost $\frac{2}{3}$ of $1\frac{1}{5}$ dollars, what will 1 orange cost? *Ans.* $\frac{1}{10}$ of a dollar.

23. A man bought $\frac{7}{9}$ of a farm and sold $\frac{2}{3}$ of his share; what part of the whole farm did he sell? what part had he left? *Ans.* Sold $\frac{14}{27}$.

24. If a barrel of sugar is worth 22 dollars, what is $\frac{7}{10}$ of it worth? *Ans.* $15\frac{2}{5}$ dollars.

25. How many hours will it take a man to travel 136 miles, if he travel $3\frac{2}{7}$ miles an hour? *Ans.* $41\frac{9}{23}$ hours.

26. How many barrels of apples can be bought for 18 dollars, at $1\frac{3}{16}$ dollars a barrel? *Ans.* $15\frac{3}{19}$ barrels.

27. If the smaller of two fractions be $\frac{4}{11}$, and the difference $\frac{2}{5}$, what is the greater? *Ans.* $\frac{42}{55}$.

28. If the sum of two fractions is $1\frac{1}{8}$, and one of them is $\frac{9}{20}$, what is the other? *Ans.* $\frac{27}{40}$.

29. If the dividend be $\frac{20}{21}$, and the quotient $\frac{5}{7}$, what is the divisor? *Ans.* $1\frac{1}{3}$.

30. If the divisor be $\frac{9}{13}$, and the quotient $3\frac{1}{3}$, what is the dividend? *Ans.* $2\frac{4}{13}$.

31. How many bushels of oats worth $\frac{2}{5}$ of a dollar a bushel, will pay for $\frac{2}{3}$ of a barrel of flour worth 9 dollars a barrel? *Ans.* 15 bushels.

32. At $\frac{3}{7}$ of a dollar a rod, what will it cost to dig $\frac{1}{2}$ of $\frac{3}{4}$ of $5\frac{1}{2}$ rods of ditch? *Ans.* $\frac{99}{112}$ dollars.

33. If a man has $24\frac{7}{8}$ bushels of clover seed, and he sells $\frac{3}{4}$ of it, how much has he left? *Ans.* $6\frac{7}{32}$ bushels.

34. A man had 6 lots of land, each containing $37\frac{3}{4}$ acres; how many acres did they all contain?

35. If $\frac{5}{6}$ of a ton of hay can be bought for 15 dollars, what part of a ton can be bought for 1 dollar?

DECIMAL FRACTIONS.

NOTATION AND NUMERATION.

103. **Decimal Fractions** are fractions which have for their denominator 10, 100, 1000, or 1 with any number of ciphers annexed.

Decimal fractions are commonly called *decimals.*

Since $\frac{1}{10}=\frac{10}{100}$, $\frac{1}{100}=\frac{10}{1000}$, &c., the denominators of decimal fractions increase and decrease in a tenfold ratio, the same as simple numbers.

104. In the formation of Decimals a unit is divided into 10 equal parts, called *tenths;* each of these *tenths* is divided into 10 other equal parts called *hundredths;* each of these *hundredths* into 10 other equal parts, called *thousandths;* and so on. Since the denominators of decimal fractions increase and decrease by the scale of 10, the same as simple numbers, in writing decimals the denominators may be omitted.

105. The Decimal sign (.) is always placed before decimal figures to distinguish them from integers. It is commonly called the *decimal point.* Thus,

$\frac{6}{10}$ is expressed .6
$\frac{54}{100}$ " " .54
$\frac{279}{1000}$ " " .279

.5 is 5 tenths, which $=\frac{1}{10}$ of 5 units;
.05 is 5 hundredths, " $=\frac{1}{10}$ of 5 tenths;
.005 is 5 thousandths, " $=\frac{1}{10}$ of 5 hundredths.

And universally, the value of a figure in any decimal place is $\frac{1}{10}$ the value of the same figure in the next left hand place.

106. The relation of decimals and integers to each other is clearly shown by the following

DECIMAL NUMERATION TABLE.

Millions.	Hund. of thousands.	Tens of thousands.	Thousands.	Hundreds.	Tens.	Units.	*Decimal Sign.*	Tenths.	Hundredths.	Thousandths.	Ten-thousandths.	Hundred-thousandths.	Millionths.
5	7	3	2	7	5	4	.	5	7	3	2	5	6

By examining this table we see that

Tenths	are expressed by	one	figure.
Hundredths	"	"	" two figures.
Thousandths	"	"	" three "

107. Since the denominator of tenths is 10, of hundredths 100, of thousands 1000, and so on, a decimal may be expressed by writing the numerator only; but in this case the numerator or decimal must always contain as many decimal places as are equal to the number of ciphers in the denominator; and the denominator of a decimal will always be the unit, 1, with as many ciphers annexed as are equal to the number of figures in the decimal or numerator.

The decimal point must never be omitted.

EXAMPLES FOR PRACTICE.

1. Express in figures seven-tenths. *Ans.* .7.
2. Write twenty-five hundredths. *Ans.* .25.
3. Write nine hundredths. *Ans.* .09.
4. Write one hundred twenty-five thousandths.
5. Write eighteen thousandths.

6. Write fifty-eight hundredths.

7. Write two hundred thirty-six thousandths.

8. Write one thousand three hundred twenty ten-thousandths. *Ans.* .1320.

9. Write seven hundred thirty-two ten-thousandths.

Read the following decimals:

.06	.143	.000	.479
.34	.037	.3240	.00341
.80	.472	.1026	.102367

108. A mixed number is a number consisting of integers and decimals; thus, 71.406 consists of the integral part, 71, and the decimal part, .406; it is read the same as $71\frac{406}{1000}$, 71 and 406 thousandths.

EXAMPLES FOR PRACTICE.

1. Write twenty-four, and four tenths. *Ans.* 24.4.

2. Write thirty-two, and five hundredths,

3. Write seventy-six, and forty-six thousandths.

4. Write one hundred twelve, and one hundred ninety thousandths. *Ans.* 112.190.

5. Write sixty-three, and forty-four ten-thousandths.

6. Write seventy-five, and one hundred forty ten-thousandths.

7. Write five, and 5 hundred thousanths.

8. Write sixteen, and 21 ten-thousandths.

9. Write eight, and 234 hundred thousandths.

10. Write forty, and 75 hundred thousandths.

Ans. 40.00075.

11. Read the following numbers:

42.08	50.002	640.00010
81.110	161.0301	7.4230
120.0342	14.42000	3.01206

109. From the foregoing explanations and illustrations we derive the following important

PRINCIPLES OF DECIMAL NOTATION AND NUMERATION.

1. The value of any decimal figure depends upon its *place* from the decimal point; thus .3 is ten times .03.

2. Prefixing a cipher to a decimal decreases its value the same as dividing it by ten; thus, .03 is $\frac{1}{10}$ the value of .3.

3. Annexing a cipher to a decimal does not altar its value, since it does not change the *place* of the significant figures of the decimal; thus, $\frac{6}{10}$, or, .6, is the same as $\frac{60}{100}$, or .60.

4. Decimals increase from right to left, and decrease from left to right, in a tenfold ratio; and therefore they may be added, subtracted, multiplied, and divided the same as whole numbers.

5. The denominator of a decimal, though never expressed, is always the unit 1, with as many ciphers annexed as there are figures in the decimal.

6. To read decimals requires two numerations; first, *from* units, to find the name of the denominator, and second, *towards* units, to find the value of the numerator.

110. Having analyzed all the principles upon which the writing and reading of decimals depend, we will now present these principles in the form of rules.

RULE FOR DECIMAL NOTATION.

I. *Write the decimal the same as a whole number, placing ciphers where necessarg to give each significant figure its true local value.*

II. *Place the decimal point before the first figure.*

RULE FOR DECIMAL NUMERATION.

I. *Numerate from the decimal point, to determine the denominator.*

II. *Numerate towards the decimal point, to determine the numerator.*

III. *Read the decimal as a whole number, giving it the name of its lowest decimal unit, or right hand figure.*

EXAMPLES FOR PRACTICE.

1. Write 325 ten-thousandths. *Ans.* .0325.
2. Write four hundred ten-thousandths.
3. Write 117 ten-thousandths.
4. Write ten ten-thousandths. *Ans.* .0010.
5. Write 250 millionths. *Ans.* .000250.
6. Write twelve hundred ten-thousandths.
7. Write 9 hundred-thousandths. *Ans.* .00009.
8. Read the following decimals.

.1236	.00061	.32760
.0080	.720000	040721

9. Write four hundred, and nine tenths.

Ans. 400.9.

10. Write twenty-seven, and fifty-six hundredths.
11. Write eighty-five, and one hundred fifty thousandths.
12. Write one thousand, and twelve millionths.
13. Write three hundred sixty-five, and one thousand eight hundred seven hundred-thousandths.

Ans. 365.01807.

14. Write nine hundred ninety, and three thousand two hundred fourteen millionths. *Ans.* 990.003214.
15. Read the following numbers:

71.03	11.0003	34.800000
126.326	240.01376	9.1263476

REDUCTION.

CASE I.

111. To reduce decimals to a common denominator.

1. Reduce .3, .09, .0426, .214 to a common denominator.

OPERATION.

.3000
.0900
.0426
.2140

ANALYSIS. A common denominator must contain as many decimal places as is equal to the greatest number of decimal figures in any of the given decimals. We find that the third number contains four decimal places, and hence 10000 must be a common denominator. As annexing ciphers to decimals does not alter their value, we give to each number four decimal places, by annexing ciphers, and thus reduce the given decimals to a common denominator. Hence,

RULE. *Give to each number the same number of decimal places, by annexing ciphers.*

EXAMPLES FOR PRACTICE.

2. Reduce .7, .073, .42, .0020 and .007 to a common denominator.

3. Reduce .004, .00032, .6, .37 and .0314 to a common denominator.

4. Reduce 1 tenth, 46 hundredths, 15 thousandths, 462 ten-thousandths, and 28 hundred-thousandths, to a common denominator.

5. Reduce 9 thousandths, 9 ten-thousandths, 9 hundred-thousandths and 9 millionths to a common denominator.

6. Reduce 42.07, 102.006, 7.80, 400.01234 to a common denominator.

7. Reduce 300.3, 8.1003, 14.12614, 210.000009, and 1000.02 to a common denominator.

CASE II.

112. To reduce a decimal to a common fraction.

1. Reduce .125 to an equivalent common fraction.

OPERATION. $.125=\frac{125}{1000}=\frac{1}{8}$

ANALYSIS. Writing the decimal figures, .125, over the common denominator, 1000, we have $\frac{125}{1000}=\frac{1}{8}$.

RULE. *Omit the decimal point, supply the proper denominator, and then reduce the fraction to its lowest terms.*

EXAMPLES FOR PRACTICE.

1. Reduce .08 to a common fraction. *Ans.* $\frac{2}{25}$.
2. Reduce .625 to a common fraction. *Ans.* $\frac{5}{8}$.
3. Reduce .375 to a common fraction. *Ans.* $\frac{3}{8}$.
4. Reduce .008 to a common fraction. *Ans.* $\frac{1}{125}$.
5. Reduce .4 to a common fraction. *Ans.* $\frac{2}{5}$.
6. Reduce .024 to a common fraction. *Ans.* $\frac{3}{125}$

CASE III.

113. To reduce a common fraction to a decimal.

2. Reduce $\frac{3}{4}$ to its equivalent decimal.

OPERATION.

4)3.0(7 tenths.
2.8

4)20(5 hundredths.
20

— *Ans.* .75.

or 4)3.00

.75 *Ans.*

ANALYSIS. Since we can not divide the numerator 3, by 4, we reduce it to *tenths* by annexing a cipher, and then dividing we obtain 7 tenths, and a remainder of 2 tenths. Reducing this remainder to *hundredths* by annexing a cipher, and dividing by 4, we obtain 5 hundredths. The sum of the quotients gives .75, the required answer.

RULE I. *Annex ciphers to the numerator, and divide by the denominator.*

II. *Point off as many decimal places in the result as are equal to the number of ciphers annexed.*

EXAMPLES FOR PRACTICE.

1. Reduce $\frac{1}{2}$ to a decimal. *Ans.* .5.
2. Reduce $\frac{1}{4}$ to a decimal. *Ans.* .25.
3. Reduce $\frac{2}{5}$ to a decimal. *Ans.* .4.
4. Reduce $\frac{4}{5}$ to a decimal. *Ans.* .8.
5. Reduce $\frac{1}{8}$ to a decimal. *Ans.* .125.
6. Reduce $\frac{9}{10}$ to a decimal. *Ans.* .9.
7. Reduce $\frac{5}{8}$ to to a decimal. *Ans.* .625.
8. Reduce $\frac{1}{25}$ to a decimal. *Ans.* .04.
9. Reduce $\frac{5}{16}$ to a decimal. *Ans.* .3125.
10. What decimal is equivalent to $\frac{17}{20}$? *Ans.* .85.
11. What decimal is equivalent to $\frac{3}{16}$? *Ans.* .1875.
12. What decimal is equivalent to $\frac{2}{125}$? *Ans.* .016.

ADDITION.

114. Since the same law of local value extends both to the right and left of units' place; that is, since decimals and simple integers increase and decrease uniformly by the scale of ten, it is evident that decimals may be added, subtracted, multiplied and divided in the same manner as integers.

1. What is the sum of 4.314, 36.42, 120.0042, and .4276?

OPERATION.

4.314
36.42
120.0042
.4276
161.1658

ANALYSIS. We write the numbers so that the figures of like orders of units shall stand in the same columns; that is, units under units, tenths under tenths, hundredths under hundredths, &c. This brings the decimal points directly under each other. Commencing at the right hand, we add each column separately, and carry as in whole numbers, and in the result we place a decimal point between units and tenths, or directly under the decimal point in the numbers added From this example we derive the following

RULE. I. *Write the numbers so that the decimal points shall stand directly under each other.*

II. *Add as in whole numbers, and place the decimal point, in the result, directly under the points in the numbers added.*

EXAMPLES FOR PRACTICE.

2. What is the sum of 2.7, 30.84, 75.1, 126.414 and 3.06? *Ans.* 238.114.

3. What is the sum of 1.7, 4.45, 6.75, 1.705, .50 and .05? *Ans.* 15.155.

4. Add 105.7, 19.4, 1119.05, 648.006 and 19.041. *Ans.* 1911.197.

5. Add 48.1, .0481, 4.81, .00481, 481. *Ans.* 533.96291.

6. Add 1.151, 13.29, 116.283, 9.0275 and .61. *Ans.* 140.3615.

7. Add .8, .087, .626, .8885 and .49628.

8. What is the sum of 91.003, 16.4691, 160.00471, 700.05, 900.0006, .0315? *Ans.* 1867.55891.

9. What is the sum of fifty-four, and thirty-four hundredths; one, and nine ten-thousandths; three, and two hundred seven millionths; twenty-three thousandths; eight, and nine tenths; four, and one hundred thirty-five thousandths? *Ans.* 71.399107.

10. How many acres of land in four farms, containing respectively, 61.843 acres, 120.75 acres, 142.4056 acres, and 180.750 acres? *Ans.* 505.7486.

11. How many yards of cloth in 3 pieces, the first containing $21\frac{1}{2}$ yards, the second $36\frac{3}{4}$ yards, and the third 40.15 yards? *Ans.* 98.40.

12. A man owns 4 city lots, containing $32\frac{1}{4}$, $36\frac{3}{5}$, $40\frac{5}{8}$, 42.73 rods of land respectively; how many rods in all? *Ans.* 152.205 rods.

SUBTRACTION.

115. From 124.2750 take 47.3126.

OPERATION.

124.2750
47.3126
Ans. 76.9624

ANALYSIS. Write the subtrahend under the minuend, placing units under units, tenths under tenths, &c. Commencing at the right hand, we subtract as in whole numbers, and in the remainder we place the decimal point directly under those in the numbers above. If the number of decimal places in the minuend and subtrahend are not equal, they may be reduced to the same number of decimal places before subtracting, by annexing ciphers. Hence the

RULE I. *Write the numbers so that the decimal points shall stand directly under each other.*

II. *Subtract as in whole numbers, and place the decimal point in the result directly under the points in the given numbers.*

EXAMPLES FOR PRACTICE.

	(2)	(3)	(4)
Minuend,	12.07	37.4562	.003476
Subtrahend,	4.3264	.97	.375
Remainder,	7.7436	36.4862	.628476

5. From 463.05 take 17.0613. *Ans.* 445.9887.
6. From 134.63 take 101.1409. *Ans.* 33.4891.
7. From 189.6145 take 10.151. *Ans.* 179.4635.
8. From 671.617 take 116.1. *Ans* 555.517.
9. From 480. take 245.0075. *Ans.* 234.9925.
10. Subtract .09684 from .145. *Ans.* .04816.
11. Subtract .2371 from .2754. *Ans.* .0383.
12. Subtract 215.7 from 271. *Ans.* 55.3.
13. Subtract .0007 from 107. *Ans.* 106.9993.
14. Subtract 1.51679 from 27.15. *Ans.* 25.63321.

15. Subtract $37\frac{1}{2}$ from 84.125. *Ans.* 46.625.

16. Subtract $3\frac{3}{4}$ from 9.3261. *Ans.* 5.5761.

17. Subtract 25.072 from $112\frac{5}{8}$. *Ans.* 87.553.

18. A man owned fifty-four hundredths of a township of land, and sold fifty-four thousandths of the same, how much did he still own? *Ans.* .486.

19. From 10 take three millionths. *Ans.* 9.999997.

20. A man owning 475 acres of land, sold at different times 80.75 acres, $100\frac{1}{8}$ acres, and 125.625 acres; how much land had he left? *Ans.* 168.5 acres.

MULTIPLICATION.

116. 1. What is the product of .25 multiplied by .5.

OPERATION.

.25
.5

.125 *Ans.*

ANALYSIS. We first multiply as in whole numbers; then, since the multiplicand has 2 decimal places and the multiplier 1, we point off 2+1=3 decimal places in the product. The reason for this will be evident, by considering both factors common fractions, and then multiplying as in (**99**), thus: $.25=\frac{25}{100}$, and $.5=\frac{5}{10}$; and $\frac{25}{100}\times\frac{5}{10}=\frac{125}{1000}$, which written decimally is .125 *Ans.* Hence the

RULE. *Multiply as in whole numbers, and from the right hand of the product point off as many figures for decimals as there are decimal places in both factors.*

NOTES. 1. If there be not as many figures in the product as there are decimals in both factors, supply the deficiency by prefixing ciphers.

2. To multiply a decimal by 10, 100, 1000, &c., remove the point as many places to the right as there are ciphers on the right of the multiplier.

EXAMPLES FOR PRACTICE.

(2)	(3)	(4)
.241	9.4263	.01346
.7	.5	.06
.1687	4.71315	.0008076

5. Multiply 7.1 by 8.2. *Ans.* 58.22.
6. Multiply 15.5 by .08. *Ans.* 1.24.
7. Multiply 8.123 by .09. *Ans.* .73107.
8. Multiply 4.5 by .15. *Ans.* .675.
9. Multiply 450. by .02. *Ans.* 9.
10. Multiply 341.45 by .007. *Ans.* 2.39015.
11. Multiply 3020. by .015. *Ans.* 45.3.
12. Multiply .132 by .241. *Ans.* .031812.
13. Multiply .23 by .009. *Ans.* .00207.
14. Multiply 7.02 by 5.27. *Ans.* 36.9954.
15. Multiply .004 by .04. *Ans.* .00016.
16. Multiply 2461. by .0529. *Ans.* 130.1869.
17. Multiply .007853 by .035. *Ans.* .000274855.
18. Multiply 25.238 by 12.17. *Ans.* 307.14646.
19. Multiply .3272 by 10. *Ans.* 3.272.
20. Multiply .3272 by 100. *Ans.* 32.72.
21. Multiply .3272 by 1000. *Ans.* 327.2.
22. Find the value of $.25 \times .5 \times 12$. *Ans.* 1.5.
23. Find the value of $.07 \times 2.4 \times .015$. *Ans.* 00252.
24. Find the value of $6\frac{1}{2} \times .8 \times 3.16$. *Ans.* 16.432.

25. If a man travel 3.75 miles an hour, how far will he travel in 9.5 hours? *Ans.* 35.626 miles.

26. If a sack of salt contain 94.16 pounds, how many pounds will 17 such sacks contain?

Ans. 1600.72 pounds.

27. If a man spend .87 of a dollar in 1 day, how much will he spend in 15.525 days?

Ans. 13.50675 dollars.

28. One rod is equal to 16.5 feet; how many feet in 30.005 rods? *Ans.* 495.0825.

29. How many gallons of molasses in .54 of a barrel, there being 31.5 gallons in 1 barrel? *Ans.* 17.01 gallons.

DIVISION.

117. 1. What is the quotient of .225 divided by .5?

OPERATION.

.5).225

.45 *Ans.*

ANALYSIS. We perform the division the same as in whole numbers, and the only difficulty we meet with is in pointing off the decimal places in the quotient. To determine how many places to point off, we may reduce the decimals to common fractions, thus; $.225=\frac{225}{1000}$ and $5=\frac{5}{10}$, performing the division as in (97), we have $\frac{225}{1000}\div\frac{5}{10}=\frac{225}{1000}\times\frac{10}{5}=\frac{45}{100}$; and this quotient expressed decimally, is .45. Here we see that the dividend contains as many decimal places as are contained in both divisor and quotient. Hence the following

RULE. *Divide as in whole numbers, and from the right hand of the quotient point off as many places for decimals as the decimal places in the dividend exceed those in the divisor.*

NOTES. 1. If the number of figures in the quotient be less than the excess of the decimal places in the dividend over those in the divisor, the deficiency must be supplied by prefixing ciphers.

2. If there be a remainder after dividing the dividend, annex ciphers, and continue the division; the ciphers annexed are decimals of the dividend.

3. The dividend must always contain at least as many decimal places as the divisor, before commencing the division.

4. In most business transactions, the division is considered sufficiently exact when the quotient is carried to 4 decimal places, unless great accuracy is required.

5. To divide by 10, 100, 1000, &c., remove the decimal point as many places to the left as there are ciphers on the right hand of the divisor.

EXAMPLES FOR PRACTICE.

(2)	(3)	(4)	(5)
.6).426	.8)3.7624	.05)81.60	.009).00207
.71.	4.703	1632.	.23

(6)	(7)	(8)
.075).9375(12.5	.288)18.0000(.0625	.0025)15.875(6350.
75	1728	150
187	720	87
150	576	75
375	1440	125
375	1440	125

9. Divide 44 by .4. *Ans.* 110.
10. Divide 15 by .25. *Ans.* 60.
11. Divide .3276 by .42. *Ans.* .78.
12. Divide .00288 by .08. *Ans.* .036.
13. Divide .0992 by .32. *Ans.* .31.
14. Divide 17.6 by 44. *Ans.* .5.
15. Divide .0000021 by .0007. *Ans.* .003.
16. Divide .56 by 1.12. *Ans.* 5.
17. Divide 1496.04 by 10. *Ans.* 149.604.
18. Divide 1496.04 by 100. *Ans.* 14.9604.
19. Divide 1596.04 by 1000. *Ans.* 1.49604.
20. Divide 4.96 by 100. *Ans.* .0496.
21. Divide 10 by .1. *Ans.* 100.
22. Divide 100 by .2. *Ans.* 500.

23. If 2.5 acres produce 34.75 bushels of wheat, how much does one acre produce? *Ans.* 13.9 bushels.

24. If a man travels 21.4 miles a day, how many days will he require to travel 461.03 miles?

25. If a man build 812.5 rods of fence in 100 days, how many rods does he build each day?

26. Paid 131.15 for 61 sheep; how much was paid for each? *Ans.* 2.15 dollars.

PROMISCUOUS EXAMPLES.

1. Add twenty-five hundredths, six hundred fifty-four thousandths, one hundred and ninety-nine thousandths, and seven thousand five hundred sixty-nine ten-thousandths.

Ans. 1.8599.

2. From ten take ten thousandths. *Ans.* 9.99.

3. What is the difference between forty thousand, and forty thousandths? *Ans.* 39999.960.

4. Multiply sixty-five hundredths, by nine hundredths.

Ans. .0585.

5. Divide 324 by 6400. *Ans.* .050625.

6. Reduce .125 to a common fraction. *Ans.* $\frac{1}{8}$.

7. Reduce $\frac{7}{8}$ to a decimal fraction. *Ans.* .875.

8. Divide .016004 by .004. *Ans.* 4.001.

9. Reduce $\frac{17}{25}$ to a decimal fraction. *Ans.* .68.

10. Reduce .4, .007, .1142, .036, .00015, and .42, to a common denominator.

11. At 13.9 dollars a ton, what will 2.5 tons of hay cost?

Ans. 34.75 dollars.

12. If a pound of sugar cost .09 dollars, how many pounds can be bought for 5.85 dollars? *Ans.* 65 pounds.

13. If 40.02 bushels of potatoes are raised upon 1 acre of land, how many acres would be required to raise 4580.64 bushels? *Ans.* 114.458 acres.

14. At 11 dollars a ton, how much hay can be bought for 13.75 dollars? *Ans.* 1.25 tons.

15. If a man travel 32.445 miles in a day, how far can he travel in .625 of a day? *Ans.* 20.278125 miles.

16. If 2 pounds of sugar cost .1875 dollars, what will be the cost of 10 pounds? *Ans.* .9375 dollars.

17. If 3 barrels apples cost 19.125 dollars, what will be the cost of 100 barrels? *Ans.* 337.5 dollars.

UNITED STATES MONEY.

118. **United States Money** is the legal currency of the United States, and was established by act of Congress August 8, 1786. Its denominations and their relative values are shown in the following

TABLE.

10 mills (*m.*)	make 1 cent,	c.
10 cents	" 1 dime,	d.
10 dimes	" 1 dollar,	$.
10 dollars	" 1 eagle,	E.

NOTE.—The currency of the United States is decimal currency, and is sometimes called *Federal Money.*

119. The character, $, before any number indicates that it expresses United States money. Thus $75 expresses 75 dollars.

120. The dollar is the *unit* of United States money; dimes, cents, and mills are fractions of a dollar, and are separated from the dollar by the *decimal point* (.); thus, two dollars one dime two cents five mills are written $2.125.

121. By examining the above table we find

1st. That the dollar being the unit, dimes, cents and mills are respectively tenths, hundredths and thousandths of a dollar.

2d. That the denominations of United States money increase and decrease the same as simple numbers and decimals, and are expressed according to the decimal system of notation.

Hence we conclude that

United States money may be added, subtracted, multiplied and divided in the same manner as decimals.

Dimes are not read as *dimes*, but the two places of dimes and cents are appropriated to cents; thus 1 dollar 3 dimes 2 cents, or $1.32, are read one dollar thirty-two cents; hence,

When the number of cents is less than 10, we *write a cipher before it in the place of dimes.*

NOTE. The half cent is frequently written as 5 mills; thus, 24½ cents, written $.245.

EXAMPLES FOR PRACTICE.

1. Write five dollars twenty-five cents. *Ans.* $5.25.
2. Write four dollars eight cents. *Ans.* $4.08.
3. Write twelve dollars thirty-six cents.
4. Write seven dollars sixteen cents.
6. Write ten dollars ten cents.
7. Write sixty-five cents four mills. $.654.
8. Write one dollar five cents eight mills. $1.058.
9. Write eighty-seven cents five mills. *Ans.* $.875.
10. Write one hundred dollars one cent one mill. *Ans.* $100.011.
11. Read $4.07; $3.094; $10.50; $25.02.

REDUCTION.

122. 1. How many cents are there in 75 dollars?

OPERATION.

75
100
7500 cents.

ANALYSIS. Since in 1 dollar there are 100 cents, in 75 dollars there are 75 times 100 cents or 7500 cents. To multiply by 10, 100, &c., we annex as many ciphers to the multiplicand as there are ciphers in the multiplier, (**62**). Hence

To change dollars to cents, multiply by 100; *that is, annex* TWO *ciphers.* And

To change dollars to mills, annex THREE *ciphers.*

To change cents to mills, annex ONE *cipher.*

EXAMPLES FOR PRACTICE.

2. Reduce \$24 to cents. *Ans.* 2400 cents.
3. Reduce \$42 to cents. *Ans.* 4200 cents.
4. Reduce \$14 to mills. *Ans.* 14000 mills.
5. Reduce \$102 to cents.
6. Change \$35 to mills.
7. Change 66 cents to mills. *Ans.* 660 mills.
8. Change 73 cents to mills.

NOTE. To change dollars and cents, or dollars, cents, and mills to mills, remove the decimal point and sign, \$.

9. Change \$4.28 to cents. *Ans.* 428 cents.
10. Change \$18.07 to cents. *Ans.* 1807 cents.
11. Change \$6.325 to mills. *Ans.* 6325 mills.
12. In \$7.01 how many cents?
13. In 94 cents how many mills?
14. In \$51 how many cents?

1. In 3427 cents how many dollars?

OPERATION.

1/00)34/27

\$34.27 *Ans.*

ANALYSIS. Since 100 cents equal 1 dollar, 3427 cents equal as many dollars as 100 is contained times in 3427, which is 34.27 times. To divide by 10, 100, &c., cut off as many figures from the right of the dividend as there are ciphers in the divisor, (72). Hence

To change cents to dollars, divide by 100; *that is, point off* TWO *figures from the right.* And

To change mills to dollars, point off THREE *figures.*

To change mills to cents, point off ONE *figure.*

EXAMPLES FOR PRACTICE.

2. Change 972 cents to dollars. *Ans.* \$9.72.
3. Change 1609 cents to dollars. *Ans.* \$16.09.
4. Change 3476 mills to dollars *Ans.* \$3.476

5. In 34671 cents how many dollars?
6. 10307 cents how many dollars?
7. In 203062 mills how many dollars? *Ans.* $203.062.
8. Reduce 672 mills to cents. *Ans.* $.672.
9. Reduce 3104 mills to dollars.
10. Reduce 17826 cents to dollars.

ADDITION.

123. 1. What is the sum of $12.50, $8.125, $4.076, $15.375 and $22?

OPERATION.

$12.50
8.125
4.076
15.375
22.000

$62.076 *Ans.*

ANALYSIS. Writing dollars under dollars, cents under cents, &c., so that the decimal points shall stand under each other, we add and point off as in addition of decimals. Hence the following

RULE. I. *Write dollars under dollars, cents under cents, &c.*

II. *Add as in simple numbers, and place the point in the amount as in addition of decimals.*

EXAMPLES FOR PRACTICE.

(2)	(3)	(4)	(5)
$ 42.64	$100.375	$750.00	$1042.875
126.085	13.09	140.07	427.035
304.127	65.82	35.178	50.50
14.42	400.00	6.004	7.08

6. What is the sum of 30 dollars 9 cents; 200 dollars 63 cents; 27 dollars 36 cents 4 mills, and 10 dollars 16 cents?
Ans. $268.244.

7. Add 390 dollars 37 cents 5 mills, 187 dollars 50 cents, 90 dollars 5 cents 5 mills, and 400 dollars 40 cents.
Ans. $1068.33.

8. A lady paid \$45.40 for some furs, \$12.375 for a dress, \$5 for a bonnet and \$1.125 for a pair of gloves; how much did she pay for all?

9. A farmer sold a cow for \$20, a horse for \$96.50, a yoke of oxen for \$66.875, and a ton of hay for \$9.40; how much did he receive for all? *Ans.* \$192.775.

10. Bought a hat for \$4.50, a pair of boots for \$5.62½, an umbrella for \$2.12½, and a pair of gloves for \$.87½; what was the cost of the whole? *Ans.* \$13.125.

11. A grocer bought a barrel of sugar for \$17.84, a box of tea for \$36.12½, a cheese for \$4, and a tub of butter for \$7.09; what was the cost of all?

12. A merchant bought a quantity of goods for \$458.25, paid for duties \$45; for freights \$98.62½, and for insurance \$16.40; how much was the whole cost?

Ans. \$618.275.

13. Bought some sugar for \$1.75, some tea for \$.90, some butter for \$2.12½, some eggs for \$.37½, and some spice for \$.25; what was the cost of the whole? *Ans.* \$5.40.

14. Paid for building a house \$1945.75, for painting the same \$275.60, for furniture \$648.87½, and for carpets \$105.10; what was the cost of the house and furnishing?

Ans. \$2075.325.

15. A farmer receives 120 dollars 45 cents for wheat, 36 dollars 62½ cents for corn, 14 dollars 9 cents for potatoes, and 63 dollars for oats; how much does he receive for the whole?

16. A lady who went shopping, bought a dress for 7 dollars 27 cents, trimmings for 87½ cents, some tape for 6 cents, some thread for 12½ cents, and some needles for 9 cents; how much did she pay for all? *Ans.* \$8.42.

SUBTRACTION.

124. 1. From 246 dollars 82 cents 5 mills, take 175 dollars 27 cents.

OPERATION.

$246.825
175.27
$71.555 *Ans.*

ANALYSIS. Writing the less number under the greater, dollars under dollars, cents under cents, &c., we subtract and point off in the result as in subtraction of decimals. Hence

RULE. I. *Write the subtrahend under the minuend, dollars under dollars, cents under cents, &c.,*

II. *Subtract as in simple numbers, and place the point in the remainder as in subtraction of decimals.*

EXAMPLES FOR PRACTICE.

	(2)	(3)	(4)	(5)
From	$125.05	$327.105	$112.000	$43.375
Take	43.278	100.09	.875	2.06
Ans.	$81.772	$227.015	$111.125	$41.315

6. From $3472.50 take $1042.125. *Ans.* $2430.375.
7. From $540 take $256.67. *Ans.* $283.33.
8. From $82.04 take $80.625. *Ans.* $1.415.
9. From 3 dollars 10 cents, take 75 cents. *Ans.* $2.35.
10. From 10 dollars, take 5 dollars 10 cts. *Ans.* $4.90.
11. From 100 dollars, take 50 dollars 50 cents.
12. From 1001 dollars 9 cents, take 300 dollars.
13. From 2 dollars, take 75 cents. *Ans.* $1.25.
14. From 96 cents, take 12½ cents. *Ans.* $.835.
15. From 1 dollar take 25 cents. *Ans.* $.75.
16. From 50 cents take 37 cents 5 mills. *Ans.* $.125.
17. From 5 dollars, take 50 cents 8 mills. *Ans.* $4.492.
18. From 4 dollars, take 1 dollar 40 cents 5 mills.
19. Sold a horse for $200, which was $45.50 more than he cost me; how much did he cost me? *Ans.* $154.50

20. A man bought a farm for $4640, and sold it for $5027.50; how much did he gain? *Ans.* $387.50.

21. Borrowed $25 and returned $15.60; how much remained unpaid? *Ans.* 9.40.

22. A merchant having $10475, paid $2426 for a store, and $5327.875 for goods; how much money had he left?

Ans. $2721.125.

23. Bought a sack of flour for $3.12½; how much change must I receive for a 5 dollar bill? *Ans.* $1.875.

24. Bought groceries to the amount of $1.875; how much change must I receive for a 2 dollar bill?

Ans. 12½ cents.

25. Paid $375 for a pair of horses, and sold one of them for $215.50; how much did the other one cost me?

Ans. $159.50.

26. I started on a journey with $50 and paid $10.62½ railroad fare, $7.38 stage fare, $5.96 for board and lodging, and $.75 for porterage; how much money had I left?

Ans. $25.285.

27. A farmer sold some wool for $27.16, and a ton of hay for $14.80. He received in payment a barrel of flour worth $6.875, and the remainder in money; how much money did he receive? *Ans.* $35.085.

28. A woman sold a grocer some butter for $1.48, and some eggs for $.94. She received a gallon of molasses worth 40 cents, a pound of tea worth 75 cents, and a pound of starch worth 12½ cents; how much is still her due?

Ans. $1.145.

29. A tailor bought a piece of broadcloth for $87.50, and a piece of cassimere for $62.75. He sold both pieces for $170.87½; how much did he gain on both?

Ans. $20.625.

MULTIPLICATION.

125. 1. Multiply $26.145 by 34.

OPERATION.

$26.145
34

104580
78435

$888.930 *Ans.*

ANALYSIS. We multiply as in simple numbers, always regarding the multiplier as an *abstract* number, and point off from the right hand of the result, as in multiplication of decimals. Hence the following

RULE. *Multiply as in simple numbers, and place the point in the product as in multiplication of decimals.*

EXAMPLES FOR PRACTICE.

(2)	(3)	(4)	(5)
$327.48	$82.375	$160.09	$7.875
15	46	87	123

6. What cost 8 cords of wood, at $3.50? *Ans.* $28.
7. What cost 14 barrels of flour, at $5.85 a barrel?
8. What cost 25 bushels of corn, at 75 cents a bushel?
9. At $2.125 a yard, what will 18 yards of silk cost?
10. At $.875 apiece, what will be the cost of 9 turkeys?
11. A farmer sold 40 bushels of potatoes at 37½ cents a bushel, and 21 barrels of apples at $2.25 a barrel; how much did he receive for both? *Ans.* $62.25.
11. Bought 124 acres of land at $35.75 an acre, and sold the whole for $6000; did I gain or lose, and how much? *Ans.* $1567.
13. What will be the cost of 275 bushels of oats, at 42 cents a bushel? *Ans.* $115.50.
14. A grocer bought 160 pounds of butter, at 14 cents a pound, and paid 25 pounds of tea, worth 56 cents a pound, and the remainder in cash; how much money did he pay?

15. What will be the cost of 15 yards of broadcloth, at $4.87½ a yard? *Ans.* $73.125.

16. A grocer bought a tub of butter containing 84 pounds, at 12½ cents a pound, and sold the same at 15 cents a pound; how much did he gain? *Ans.* $2.10.

17. A farmer took 3 tons of hay to market, for which he received $9.38 a ton. He bought 2 barrels of flour, at $6.94 a barrel, and 12 pounds of tea, at $.625 a pound; how much money had he left?

Ans. $6.76.

DIVISION.

126. 1. Divide $136 by 64.

64)$136.000($2.125 *Ans.*
128

80
64

160
128

320
320

ANALYSIS. We divide as in simple numbers, and as there is a remainder after dividing the dollars, we reduce the dividend to mills, by annexing three ciphers, and continue the division. Hence the following

RULE. *Divide as in simple numbers, and place the point in the quotient, as in division of decimals.*

NOTE. 1. In business transactions it is never necessary to carry the division further than to mills in the quotient.

EXAMPLES FOR PRACTICE.

(2)	(3)	(4)	(5)
5)$43.50	10)$36.00	8)$371.	12)$169.50
$8.70	$3.60	$46.375	$14.125

6. Divide \$13.75 by 11. *Ans.* \$1.25.
7. Divide \$162. by 36. *Ans.* \$4.50.
8. Divide \$246.30 by 15. *Ans.* \$16.42.
9. Divide \$1305. by 18. *Ans.* \$72.50.
10. Divide \$2.25 by 9. *Ans.* \$.25.
11. Divide \$658 by 280. *Ans.* \$2.35.
12. Divide \$195.75 by 29. *Ans.* \$6.75.
13 Divide 1388 by 100. *Ans.* \$13.88.
14. Divide \$2675.75 by 278. *Ans.* \$9.625.
15. Divide \$68 by 32. *Ans.* \$2.125.

16. Paid \$168.48 for 144 bushels of wheat; what was the price per bushel? *Ans.* \$1.17.

17. Paid \$2.80 for 35 pounds of sugar; what was the price per pound? *Ans.* \$.08.

18. If 54 cords of wood cost \$135, what is the price per cord? *Ans.* \$2.50.

19. Bought 125 bushels of oats for \$62.50; what was the cost per bushel? *Ans.* \$.50.

20. If 70 barrels of apples cost \$175, how much will 1 barrel cost? *Ans.* \$2.50.

21. If 100 acres of land cost \$3156.50, how much will be the cost of 1 acre? *Ans.* \$31.565.

22. Paid \$148.75 for 170 bushels of barley; how much was the cost per bushel? *Ans.* \$.875.

23. If 13 pounds of tea cost \$9.88, how much will 1 pound cost?

24. Bought 2500 pounds of butter for \$625; how much was the cost per pound? *Ans.* 25 cents.

25 Bought 2450 pounds of pork for \$153.12$\frac{1}{2}$; how much was the cost per pound? *Ans.* 6$\frac{1}{4}$ cents.

26. Bought 4 barrels of sugar, each containing 200 pounds, for \$72; what was the cost per pound?

PROMISCUOUS EXAMPLES.

1. A merchant bought 14 boxes of tea for $560; but it being damaged, he was obliged to sell it for $106.75 less than he gave for it; how much did he receive a box?

Ans. $32.375.

2. A farmer sold 120 bushels of wheat, at $1.12½ a bushel, and received in payment 27 barrels of flour; what did the flour cost him per barrel?

3. If 35 yards of cloth cost $122.50, how much will 29 yards cost? *Ans.* $101.50.

4. If 4 tons of coal cost $35.50, how much will 12 tons cost? *Ans.* $106.50.

5. If 29 pounds of sugar cost $3.625, how much will 15 pounds cost? *Ans.* $1.875.

6. If 12 barrels of flour cost $108, how much will 18 barrels cost? *Ans.* $162.

7. If 3 bushels of wheat cost $4.35, how much will 30 bushels cost? *Ans.* $43.50.

8. A man bought a farm containing 125 acres, for $2922.50; for how much must he sell it per acre to gain $500? *Ans.* $27.38.

9. A farmer exchanged 50 bushels of corn worth 70 cents a bushel, for 28 bushels of wheat; how much was the wheat worth a bushel. *Ans.* $1.25.

10. A person having $15000, bought 30 bales of cotton each bale containing 940 pounds, at 10 cents a pound; he next paid $6680 for a house, and then bought 1000 barrels of flour with what money he had left; how much did the flour cost him per barrel? *Ans.* $5.50.

NOTE. For a full and complete development and application of Decimals and United States money, the pupil is referred to the Author's Progressive Practical and Higher Arithmetic.

BILLS.

127. A Bill, in business transactions, is a written statement of articles bought or sold, together with the prices of each, and the whole cost.

Find the cost of the several articles, and the amount or footing of the following bills:

(1.)

CHICAGO, Sept. 20, 1861.

MR. J. C. SMITH,

Bo't. of SILAS JOHNSON,

36 pounds sugar at 8 cents a pound,	$2.88
18 pounds coffee at 15 cents a pound,	2.70
24 pounds butter at 18 cents a pound,	4.32
10 dozen eggs at 12½ cents a dozen,	1.25
4 gallons molasses at 44 cents a gallon,	1.76
Ans.	$12.91.

(2.)

ROCHESTER, Jan. 25, 1862.

JOHN DABNEY, ESQ.,

Bo't. of BARDWELL & Co.,

14 pounds coffee sugar at 11 cents a pound,	$1.54
6 pounds Y. H. tea at 62½ cents a pound,	3.75
25 pounds No. 1 mackerel at 6 cents a pound,	1.50
5 bushels potatoes at 37½ cents a bushel,	1.875
3 gallons syrup at 80 cents a gallon,	2.40
7 dozen eggs at 16 cents a dozen,	1.12
Ans.	$12.185

Received Payment,
Bardwell & Co.,
per Adams

(3.)

MEMPHIS, Aug. 20, 1862

Mr. S. P. HAILE,

Bo't of PATTERSON & Co.,

20	chests	Green Tea	at	$22.50
16	"	Black "	at	18.75
14	"	Imperial "	at	32.87½
15	sacks	Java Coffee	at	17.38
25	boxes	Oranges	at	4.62½

Received payment, $1586.575.

Patterson & Co.,

(4.)

OSWEGO, Sept. 4, 1861.

JAMES COROVAL & Co.,

Bo't. of COLLINS & SON,

12	yards	Broadcloth	at	$3.84
18	"	Cassimere	"	2.25
10	"	Satinet	"	.87½
42	"	Flannel	"	.45
35	"	Black Silk	"	1.18

$155.53.

(5.)

BOSTON, April 10, 1862.

J. G. BENNET & SON,

Bo't. of BUTLER, KING & Co.,

14	Plows	at	$10.50
8	Harrows	"	9.80
120	Shovels	"	.90
175	Hoes	"	.62½

$442.775.

COMPOUND NUMBERS.

128. A **Simple Number** is either an abstract number, or a concrete number of but one denomination. Thus, 48, 926; 48 dollars, 926 miles.

129. A **Compound Number** is a concrete number whose value is expressed in two or more different denominations. Thus, 32 dollars 15 cents; 15 days 4 hours 25 minutes.

130. A **Scale** is a series of numbers, descending or ascending, used in operations upon numbers.

NOTE. In simple numbers and decimals the scale is uniformly 10; in compound numbers the scales are varying.

CURRENCY.

I. UNITED STATES MONEY.

131. The currency of the United States is decimal currency, and is sometimes called *Federal Money*.

TABLE.

10 mills (m.)	make 1	cent,.....ct.
10 cents	" 1	dime, ...d.
10 dimes	" 1	dollar,....$.
10 dollars	" 1	eagle,.....E.

UNIT EQUIVALENTS.

		d.	ct.	m.
		1 —	1 —	10
	$	1 —	10 —	100
E	1 —	10 —	100 —	1000
1 —	10 —	100 —	1000 —	10000

SCALE—uniformly 10.

COINS. *The gold coins* are the double eagle, eagle, half eagle, quarter eagle, three-dollar piece and dollar.

The silver coins are the half and quarter dollar, dime and half dime, and three-cent piece.

The nickel coin is the cent.

II. CANADA MONEY.

132. The currency of the Canadian provinces is decimal, and the table and denominations are the same as those of the United States money.

NOTE The decimal currency was adopted by the Canadian Parliament in 1858, and the Act took effect in 1859. Previous to the latter year the money of Canada was reckoned in pounds, shillings, and pence, the same as in England.

COINS. The new Canadian coins are silver and copper.

The *silver coins* are the shilling or 20-cent piece, the dime, and half dime.

The *copper coin* is the cent.

NOTE. The 20-cent piece represents the value of the shilling of the old Canada Currency.

III. ENGLISH MONEY.

133. English or Sterling money is the currency of Great Britain.

TABLE.

4 farthings (far. or qr.)	make	1 penny,.....d.
12 pence	"	1 shilling,.....s.
20 shillings	"	1 pound or sovereign....£ or sov.

UNIT EQUIVALENTS.

		d.		far.
	s.	1	=	4
£, or sov.	1	= 12	=	48
1	= 20	= 240	=	960

SCALE—ascending, 4, 12, 20; descending, 20, 12, 4.

NOTE. Farthings are generally expressed as fractions of a penny; thus, 1 far., sometimes called 1 quarter, (qr.) =¼d.; 3 far.=¾d.

COINS. The *gold coins* are the sovereign (= £1) and the half sovereign, (= 10s.)

The *silver coins* are the crown (= 5s.), the half crown, (= 2s. 6d.), the shilling, and the 6-penny piece.

The *copper coins* are the penny, half-penny, and farthing.

WEIGHTS.

134. **Weight** is a measure of the quantity of matter a body contains, determined according to some fixed standard.

I. TROY WEIGHT.

135. **Troy Weight** is used in weighing gold, silver, and jewels; in philosophical experiments, &c.

TABLE.

24 grains (gr.) make 1 pennyweight,...pwt. or dwt.
20 pennyweights " 1 ounce, ...oz.
12 ounces " 1 pound, ...lb.

UNIT EQUIVALENTS.

		pwt.	gr.
	oz.	1 —	24
lb.	1 —	20 —	480
1 —	12 —	240 —	5760

SCALE—ascending, 24, 20, 12; descending, 12, 20, 24.

II. AVOIRDUPOIS WEIGHT.

136. **Avoirdupois Weight** is used for all the ordinary purposes of weighing.

TABLE.

16 drams (dr.) make 1 ounce,.....oz.
16 ounces " 1 pound,.....lb.
100 lb. " 1 hundred weight, .cwt.
20 cwt., = 2000 lbs., 1 ton,.....T.

UNIT EQUIVALENTS.

			oz.	dr.
		lb.	1 —	16
	cwt.	1 —	16 —	256
T. —	1 —	100 —	1600 —	25600
1 —	20 —	2000 —	32000 —	512000

SCALE—ascending, 16, 16, 100, 20; descending, 20, 100, 16, 16.

NOTE. The *long* or *gross ton*, hundred weight, and quarter were formerly in common use; but they are now seldom used except in estimating English goods at the U. S. custom-house, and in freighting and wholesaling coal from the Pennsylvania mines.

LONG TON TABLE.

28 lb.		make 1 quarter,	qr.
4 qr.	— 112 lb.	" 1 hundred weight,	cwt.
20 cwt.	— 2240 lb.	" 1 ton,	T.

The following denominations are also in use:

56 pounds make 1 firkin of butter.
196 " " 1 barrel of flour.
200 " " 1 " " beef, pork, or fish.
280 " " 1 bushel, " salt at the N. Y. State salt works
32 " " 1 " " oats.
48 " " 1 " " barley.
56 " " 1 " " corn or rye.
60 " " 1 " " wheat.

III. APOTHECARIES' WEIGHT.

137. **Apothecaries' Weight** is used by apothecaries and physicians in compounding medicines; but medicines are bought and sold by avoirdupois weight.

TABLE.

20 grains [gr.]	make 1 scruple.....sc. or ℈.	
3 scruples	" 1 dram,.....dr. or ʒ.	
8 drams	" 1 ounce,.....oz. or ℥.	
12 ounces	" 1 pound.....lb. or ℔	

UNIT EQUIVALENTS.

			sc.	gr.
		d.	1 —	20
	oz.	1 —	3 —	60
lb.	1	8 —	24 —	480
1 —	12 —	96 —	288 —	5760

SCALE—ascending, 20, 3, 8, 12; descending, 12, 8, 3.

138. COMPARATIVE TABLE OF WEIGHTS.

	Troy.	Avoirdupois.	Apothecaries.
1 pound	— 5760 grains,	— 7000 grains,	— 5760 grains,
1 ounce	— 480 "	— 437.5 "	— 480 "
	175 pounds,	— 144 pounds,	— 175 pounds.

MEASURES OF EXTENSION.

139. **Extension** has three dimensions—length, breadth, and thickness.

A **Line** has only one dimension—length.

A **Surface** or **Area** has two dimensions—length and breadth.

A **Solid** or **Body** has three dimensions—length, breadth, and thickness.

I. LONG MEASURE.

140. **Long Measure**, also called Lineal Measure, is used in measuring lines or distances.

TABLE.

12 inches (in.)	make 1 foot,.....ft.
3 feet	" 1 yard,.....yd.
$5\frac{1}{2}$ yd., or $16\frac{1}{2}$ ft.,	" 1 rod,.....rd.
40 rods	" 1 furlong,.....fur.
8 furlongs, or 320 rd.,	" 1 statute mile,.mi.

UNIT EQUIVALENTS.

| | | |

141. The following denominations are also in use:—

8	barleycorns	make 1 inch,	used by shoemakers in measuring the length of the foot.
4	inches	" 1 hand,	used in measuring the height of horses directly over the fore feet.
9	"	" 1 span.	
21.888	"	" 1 sacred cubic.	
3	feet	" 1 pace.	
6	"	" 1 fathom,	used in measuring depths at sea.
1.15	statute miles	" 1 geographic mile,	used in measuring distances at sea.
3	geographic "	" 1 league.	
60	" " or	1 degree	of latitude on a meridian or
69.16	statute " "		of longitude on the equator.
360	degrees	" the circumference of the earth.	

NOTES. 1. For the purpose of measuring cloth and other goods sold by the yard, the yard is divided into halves, quarters, fourths, eighths, and sixteenths. The old table of cloth measure is practically obsolete.

SURVEYORS' LONG MEASURE.

142. A **Gunter's Chain**, used by land surveyors, is 4 rods or 66 feet long, and consists of 100 links.

TABLE.

7.92	inches	(in.) make 1 link,	l.
25	links	" 1 rod,	rd.
4	rods, or 66 feet,	" 1 chain,	ch.
80	chains	" 1 mile,	mi.

UNIT EQUIVALENTS.

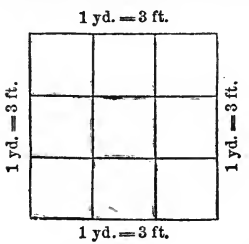
			rd.	l.	in.
			1	1 ·	7.92
		ch.	1 —	25 —	198
	mi.	1 —	4 —	100 —	792
1 —	80 —	320 —		8000 —	63360

SCALE—ascending, 7.92, 25, 4, 80; descending, 80, 4, 25, **7.92.**

NOTE. The denomination, rods, is seldom used in chain measure, distances being taken in chains and links.

II. SQUARE MEASURE.

143. A Square is a figure having four equal sides, and four equal angles or corners.



1 square yard is a figure having four sides of 1 yard or 3 feet each, as shown in the diagram. Its contents are $3 \times 3 = 9$ square feet. Hence

Thus a square foot is 12 inches long and 12 inches wide, and the contents are $12 \times 12 = 144$ square inches. A surface 20 feet long and 10 feet wide, is a rectangle, containing $20 \times 10 = 200$ square feet.

The contents or area of a square, or of any other figure having a uniform length and a uniform breadth, is found, by multiplying the length by the breadth.

144. Square Measure is used in computing areas or surfaces; as of land, boards, painting, plastering, paving, &c.

TABLE.

144	square inches (sq. in.) make	1 square foot,	sq. ft
9	square feet "	1 square yard,	sq. yd.
30¼	square yards "	1 square rod,	sq. rd.
40	square rods "	1 rood,	R.
4	roods "	1 acre,	A.
640	acres "	1 square mile,	sq. mi.

UNIT EQUIVALENTS.

				sq. ft.	sq. in.
			sq. yd.	1—	144
		sq. rd.	1—	9—	1276
	R.	1—	30¼—	272½—	39204
A.	1—	40—	1210—	10890—	1568160
sq. mi.	1— 4—	160—	4840—	43560—	6272640
1—	640— 2560—	102400—	3097600—	27878400—	40144896000

Artificers estimate their work as follows:

By the square foot: glazing and stone-cutting.

By the square yard: painting, plastering, paving, ceiling, and paper-hanging.

By the square of 100 feet: flooring, partitioning, roofing, slating, and tiling.

Brick-laying is estimated by the thousand bricks; also by the square yard, and the square of 100 feet.

NOTES. 1. In estimating the painting of moldings, cornices, etc., the measuring line is carried into all the moldings and cornices.

2. In estimating brick-laying by the square yard or the square of 100 feet, the work is understood to be 1½ bricks, or 12 inches, thick.

SURVEYORS' SQUARE MEASURE.

145. This measure is used by surveyors in computing the area or contents of land.

TABLE.

625 square links (sq. l.)	make	1 pole,.....P.
16 poles	"	1 square chain, ..sq. ch.
10 square chains	"	1 acre,.....A.
640 acres	"	1 square mile,...sq. mi.
36 square miles (6 miles square)	"	1 Township,.....Tp.

UNIT EQUIVALENTS.

			P.	sq. l.
		sq. ch.	1 —	625
	A.	1 —	16 —	1000
sq. mi.	1 —	10 —	160 —	10000
Tp. 1 —	640 —	6400 —	102500 —	64000000
1 — 36 —	23040 —	230400 —	3686400 —	2304000000

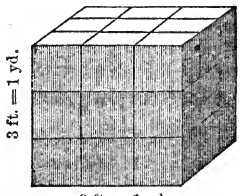
SCALE—ascending, 625, 16, 10, 630, 36; descending, 36, 640, 10, 16, 625.

NOTES. 1. A square mile of land is also called a *section*.

2. Canal and railroad engineers commonly use an engineers' chain, which consists of 100 links, each 1 foot long.

III. CUBIC MEASURE.

146. A **Cube** is a solid, or body, having **six equal** square sides or faces.



If each side of a cube be 1 yard, or 3 feet, 1 foot in thickness of this cube will contain $3\times3\times1=9$ cubic feet; and the whole cube will contain $3\times3\times3=27$ cubic feet.

A solid, or body, may have the three dimensions all alike, or all different. A body 4 ft. long, 3 ft. wide, and 2 ft. thick contains $4\times3\times2=24$ cubic or solid feet. Hence we see that

The cubic or solid contents of a body are found by multiplying the length, breadth, and thickness together.

147. Cubic Measure, also called Solid Measure, is used in estimating the contents of solids, or bodies; as timber, wood, stone, &c.

TABLE.

1728	cubic inches (cu. in.)	make 1	cubit foot, . . cu. ft.
27	cubic feet	" 1	cubic yard, cu. yd.
40	cubic feet of round timber, or	" 1	ton or load, . . . T.
50	" " hewn "		
16	cubic feet	" 1	cord foot, . . . cd. ft.
8	cord feet, or	" 1	cord of wood, . Cd.
128	cubic feet		
$24\frac{3}{4}$	cubic feet	" 1	perch of stone or masonry, Pch.

SCALE—ascending, 1728, 27, 40, 50, 16, 8, 128, $24\frac{3}{4}$; descending, $24\frac{3}{4}$, 128, 8, 16, 50, 40, 27, 1728.

NOTES. 1. A cubic yard of earth is called a load.

2. Railroad and transportation companies estimate light freight by the space it occupies in cubic feet, and heavy freight by weight.

3. A pile of wood 8 feet long, 4 feet wide, and 4 feet high, contains 1 cord; and a cord foot is one foot in length of such a pile.

4. A perch of stone or of masonry is 16½ feet long, 1½ feet wide, and 1 foot high.

MEASURES OF CAPACITY.

148. **Capacity** signifies extent of room or space.

All measures of capacity are cubic measures, solidity and capacity being referred to different units, as will be seen by comparing the tables.

Measures of capacity may be properly subdivided into two classes, Measures of Liquids, and Measures of Dry Substances.

I. LIQUID MEASURE.

149. **Liquid Measure**, also called Wine Measure, is used in measuring liquids; as liquors, molasses, water, &c.

TABLE.

4 gills (gi.)	make	1 pint, ...pt.
2 pints	"	1 quart, ...qt.
4 quarts	"	1 gallon, ...gal.
31½ gallons	"	1 barrel, ...bbl.
2 barrels, or 63 gal.	"	1 hogshead, ...hhd.

UNIT EQUIVALENTS.

					pt.		gi.
				qt.	1	—	4
			gal.	1	— 2	—	8
		bbl.	1	— 4	— 8	—	32
	hhd.	1	— 31½	— 126	— 252	—	1008
	1	— 2	— 63	— 252	— 504	—	2016

SCALE—ascending, 4, 2, 4, 31½, 2; descending, 2, 31½, 4, 2, 4.

150. The following denominations are also in use:

86 gallons		make 1 barrel	of beer.
54 "	or 1½ barrels	" 1 hogshead	" "
42 "		" 1 tierce.	
2 hogsheads, or 120 gallons,		" 1 pipe or butt.	
2 pipes, or 4 hogsheads,		" 1 tun.	

NOTES. 1. The denominations, barrel and hogshead, are used in estimating the capacity of cisterns, reservoirs, vats, &c.

2. The tierce, hogshead, pipe, butt and tun, are the names of casks, and do not express any fixed or definite measures. They are usually gauged, and have their capacities in gallons marked on them.

3. Ale or beer measure, formerly used in measuring beer, ale and milk, is almost entirely discarded.

II. DRY MEASURE.

151. **Dry Measure** is used in measuring articles not liquid; as grain, fruit, salt, roots, ashes, &c.

TABLE.

2 pints (pt.)	make 1	quart,.....qt.
8 quarts	" 1	peck,.....pk.
4 pecks	" 1	bushel,.bu. or bush.

UNIT EQUIVALENTS.

		qt.	pt.
	pk.	1 —	2
bu.	1 —	8 —	16
1 —	4 —	32 —	64

SCALE—ascending, 2, 8, 4; descending, 4, 8, 2.

NOTES. 1. In England, 8 bu. of 70 lbs each are called a *quarter*, used in measuring grain. The weight of the English quarter is ¼ of a long ton.

2. The wine and dry measures of the same denomination are of different capacities. The exact and the relative size of each may be readily seen by the following

152. COMPARATIVE TABLE OF MEASURES OF CAPACITY.

	Cu. in. in one gallon.	Cu. in. in one quart.	Cu. in. in one pint.	Cu. in. in one gill.
Wine measure,	231	$57\frac{3}{4}$	$28\frac{7}{8}$	$7\frac{7}{32}$
Dry measure,($\frac{1}{2}$pk.,)	$268\frac{4}{5}$	$67\frac{1}{5}$	$33\frac{3}{5}$	$8\frac{2}{5}$

3. The beer gallon of 282 inches is retained in use only by custom. A bushel commonly estimated at 2150.4 cubic inches.

MEASURE OF TIME.

153. Time is the measure of duration.

TABLE.

60 seconds (sec.) make 1 minute,.....min.
60 minutes " 1 hour,.....h.
24 hours " 1 day,.....da.
7 days " 1 week,.....wk.
365 days " 1 common year,.....yr.
366 days " 1 leap year,.....yr.
12 calender months " 1 year,.....yr.
100 years " 1 century,.....C.

UNIT EQUIVALENTS.

					min.	sec.
				h.	1 —	60
			da.	1 —	60 —	3600
		wk.	1 —	24 —	1440 —	86400
		1 —	7	168 —	10080 —	604800
yr.	mo.		365	8760 —	525600 —	31536000
1 —	12	—	366	8784 —	527040 —	31622400

SCALE—ascending, 60, 60, 24, 7; descending, 7, 24, 60, 60.

154. The calendar year is divided as follows :—

No. of month.	Season.	Names of months.	Abbreviations.	No. of days.
1	Winter,	January,	Jan.	31
2		February,	Feb.	28 or 29
3		March,	Mar.	31
4	Spring,	April,	Apr.	30
5		May,	——	31
6		June,	Jun.	30
7	Summer,	July,	——	31
8		August,	Aug.	31
9		September,	Sept.	30
10	Autumn,	October,	Oct.	31
11		November,	Nov.	30
12	Winter,	December,	Dec.	31
				365 or 366

NOTES. 1. The exact length of a solar year is 365 da. 5 h. 48 min. 46 sec. ; but for convenience it is reckoned 11 min. 14 sec. more than this, or 365 da. 6 h. = 365¼ da. This ¼ day, in four years makes one day, which, every fourth, bissextile, or leap year, is added to the shortest month, giving it 29 days. The leap years are exactly divisible by 4, as 1856, 1860, 1864.

The number of days in each calendar month may be easily remembered by committing the following lines :—

"Thirty days hath September,
April, June, and November ;
All the rest have thirty-one,
Save February, which alone
Hath twenty-eight ; and one day more
We add to it one year in four."

2. In most business transactions 30 days are called 1 month.

3. The centuries are numbered from the commencement of the Christian era, the months from the commencement of the year ; the days from the commencement of the month, and the hours from the commencement of the day, (12 o'clock, midnight.) Thus, May 23d, 1860, 9 o'clock A. M., is the 9th hour of the 23d day of the 5th month of the 60th year of the 19th century.

CIRCULAR MEASURE.

155. Circular Measure, or Circular Motion, is used principally in surveying, navigation, astronomy, and geography, for reckoning latitude and longitude, determining locations of places and vessels, and computing difference of time.

Each circle, great or small, is divisible into the same number of equal parts, as quarters, called quadrants, twelfths, called signs, 360ths, called degrees, &c. Consequently the parts of unequal circles, although having the same names, are of unequal lengths.

TABLE.

60 seconds (″)	make	1 minute,....′.
60 minutes	"	1 degree,....°.
30 degrees	"	1 sign,.....S.
12 signs, or 360°	"	1 circle,.....C.

UNIT EQUIVALENTS.

			′	″
		°	1 —	60
	S.	1 —	60 —	3600
C.	1 —	30 —	1800 —	108000
1 —	12 —	360 —	21600 —	1296000

SCALE—ascending, 60, 60, 30, 12; descending, 12, 30, 60, 60.

NOTES. 1. Minutes, of the earth's circumference are called geographic or nautical miles.

2. The denomination, *signs*, is confined exclusively to Astronomy.

3. A degree has no fixed linear extent. When applied to any circle, it is always $\frac{1}{360}$ part of the circumference. But, strictly speaking, it is not any part of a circle.

4. 90° make a quadrant or right-angle.

5 60° make a sextant or $\frac{1}{6}$ of a circle.

MISCELLANEOUS TABLES.

156. COUNTING.

12 units or things make 1 dozen.
12 dozen " 1 gross.
12 gross " 1 great gross.
20 units " 1 score.

157. PAPER.

24 sheets.....make....1 quire.
20 quires " 1 ream.
2 reams " 1 bundle.
5 bundles " 1 bale.

158. BOOKS.

The terms *folio, quarto, octavo, duodecimo,* &c., indicate the number of leaves into which a sheet of paper is folded.

A sheet folded in 2 leaves is called a folio.
A sheet folded in 4 leaves " a quarto, or 4to.
A sheet folded in 8 leaves " an octavo, or 8vo.
A sheet folded in 12 leaves " a 12mo.
A sheet folded in 16 leaves " a 16mo.
A sheet folded in 18 leaves " an 18mo.
A sheet folded in 24 leaves " a 24mo.
A sheet folded in 32 leaves " a 32mo.

159. COPYING.

75 words make 1 folio or sheet of common law.
90 " " 1 " " " " chancery.

160. An Aliquot Part of a number is such a part as will exactly divide that number; thus, 3, 5, 7½ are aliquot parts of 15.

NOTE. An *aliquot part* may be a whole or mixed number, while a *factor* must be a whole number.

161. ALIQUOT PARTS OF ONE DOLLAR.

50 cents $= \frac{1}{2}$ of 1 dollar.	$12\frac{1}{2}$ cents $= \frac{1}{8}$ of 1 dollar.
$33\frac{1}{3}$ cents $= \frac{1}{3}$ of 1 dollar.	10 cents $= \frac{1}{10}$ of 1 dollar.
25 cents $= \frac{1}{4}$ of 1 dollar.	$8\frac{1}{3}$ cents $= \frac{1}{12}$ of 1 dollar.
20 cents $= \frac{1}{5}$ of 1 dollar.	$6\frac{1}{4}$ cents $= \frac{1}{16}$ of 1 dollar.
$16\frac{2}{3}$ cents $= \frac{1}{6}$ of 1 dollar.	5 cents $= \frac{1}{20}$ of 1 dollar.

162. PARTS OF $1 IN NEW YORK CURRENCY.

4 shillings	$= \$\frac{1}{2}$	1 shil. 4 pence	$= \$\frac{1}{5}$
2 shillings 8d.	$= \$\frac{1}{3}$	1 shilling	$= \$\frac{1}{8}$
2 shillings	$= \$\frac{1}{4}$	6 pence	$= \$\frac{1}{16}$

163. PARTS OF $1 IN NEW ENGLAND CURRENCY.

3 shillings	$= \$\frac{1}{2}$	1 shilling	$= \$\frac{1}{6}$
2 shillings	$= \$\frac{1}{3}$	9 pence	$= \$\frac{1}{8}$
1 shillings 6d.	$= \$\frac{1}{4}$	6 pence	$= \$\frac{1}{12}$

164. ALIQUOT PARTS OF A TON.

10 hund. lbs.	$= \frac{1}{2}$ ton.	2 hund. 2 qrs.	$= \frac{1}{8}$ ton.
5 hund. lbs.	$= \frac{1}{4}$ ton.	2 hund. lbs.	$= \frac{1}{10}$ ton.
4 hund. lbs.	$= \frac{1}{5}$ ton.	1 hund. lbs.	$= \frac{1}{20}$ ton.

165. ALIQUOT PARTS OF A POUND AVOIRDUPOIS.

8 ounces	$= \frac{1}{2}$ pound.	2 ounces	$= \frac{1}{8}$ pound.
4 ounces	$= \frac{1}{4}$ pound.	1 ounce	$= \frac{1}{16}$ pound.

166. ALIQUOT PARTS OF TIME.

Parts of 1 *year.*		*Parts of* 1 *month.*	
6 months	$= \frac{1}{2}$ year.	15 days	$= \frac{1}{2}$ month.
4 months	$= \frac{1}{3}$ year.	10 days	$= \frac{1}{3}$ month.
3 months	$= \frac{1}{4}$ year.	6 days	$= \frac{1}{5}$ month.
2 months	$= \frac{1}{6}$ year.	5 days	$= \frac{1}{6}$ month.
$1\frac{1}{2}$ months	$= \frac{1}{8}$ year.	3 days	$= \frac{1}{10}$ month.
$1\frac{1}{3}$ months	$= \frac{1}{9}$ year.	2 days	$= \frac{1}{15}$ month.
1 month	$= \frac{1}{12}$ year.	1 day	$= \frac{1}{30}$ month

REDUCTION.

167. **Reduction** is the process of changing a number from one denomination to another without altering its value.

168. **Reduction Descending** is changing a number of one denomination to another denomination of *less unit value*, and is performed by multiplication; thus: \$1 = 10 dimes = 100 cents = 1000 mills; 1 yard = 3 feet = 36 inches.

1. Reduce 6 gal. 2 qt. 1 pt. to pints.

OPERATION.

6 gal. 2 qt. 1 pt.
4
—
26 qt.
2
—
Ans. 53 pt.

ANALYSIS. Since in 1 gal. there are 4 qt. in 6 gal. there 4 qt. × 6 = 24 qt. and the 2 qt. in the given number, added, makes 26 qt. in 6 gal. 2 qt. Since in 1 qt. there are 2 qt. in 26 qt. there 2 pt. × 26 = 52 pt. and the 1 pt. in the given number added, make 53 pints in the given compound number. As either factor may be used as a multiplicand, (**61**), we may consider the numbers in the descending scale as multipliers. Hence the following

RULE. I. *Multiply the highest denomination of the given compound number by that number of the scale which will reduce it to the next lower denomination, and add to the product the given number, if any, of that lower denomination.*

II. *Proceed in the same manner with the result obtained in each lower denomination, until the reduction is brought to the denomination required.*

EXAMPLES FOR PRACTICE.

2. In 8 lb. 10 oz. how many ounces? *Ans.* 138 oz.
3. In £12 6s. 9d. how many pence? *Ans.* 2961d.
4. In 4 yd. 1 ft. 10 in. how many inches?
5. In 3 mi. 5 fur. 26 rd. how many rods?

6. In 18s. 8d. 3 far. how many farthings?

Ans. 899 far.

7. Reduce 3 lb. 9 oz. 12 pwt. to pennyweights.

8. In hhd. 15 gal. 2 qt. how many pints?

9. Reduce 4 da. 5 hr. to minutes. *Ans.* 6060 min.

10. Reduce 10 bu. 1 pk. 6 qt. to pints. *Ans.* 1308 pt.

11. Reduce 14 A. 3 R. 20 sq. rd. to square rods.

12. Reduce 4 cd. 3 cd. ft. 9 cu. ft. to cubic inches.

13. Reduce 4 yr. 7 mo. to hours. *Ans.* 39600 hr.

14. Change 2 T. 11 cwt. to pounds. *Ans.* 5100 lb.

15. Change 9 lb. 9 oz. 10 pwt. to grains.

16. Change 5 lb. 6 ℥ 4 ʒ 2 ℈ 10 gr. to grains.

17. Change 3 mi. 6 fur. to feet. *Ans.* 19800 ft.

18. In 40 chains how many links? *Ans.* 4000 l.

19. In 28 sq. rd. 12 sq. yd. 4 sq. ft. how many square inches? *Ans.* 1113840 sq. in.

20. In 16 A. 4 sq. ch. 8 P. 80 sq. l. how many square links? *Ans.* 1645080 sq. l.

21. In 12 tons of round timber how many cubic inches?

22. In 8 bbl. 26 gal. how many pints? *Ans.* 2224 pt.

23. Reduce 4 pipes to quarts. *Ans.* 2016 qt.

24. Reduce 23 bu. 3 pk. to pints. *Ans.* 1520 pt.

25. Reduce 8 S. 18° 40′ to minutes. *Ans.* 15520′.

26. Reduce 15° to seconds. *Ans.* 54000″.

27. Reduce 2 months to minutes. *Ans.* 86400 min.

28. Change 2 reams 10 quires to sheets.

29. In 40 score how many single things? *Ans.* 800.

30. In 14 great gross how many dozens?

31. In 30° 20′ 24″ how many seconds?

32. In the 3 Autumn months how many hours?

33. In the three Summer months how many minutes?

34. In 75 cords how many cubic feet?

169. Reduction Ascending is changing a number of one denomination to another of *greater unit value*, and is performed by Division; thus, 1000 mills =100 cents =$1.

1. Reduce 53 pints to gallons.

OPERATION.

2)53

4)26 qt.+1 pt.

6 gal.+2 qt.

Ans. 6 gal. 2 qt. 1 pt.

ANALYSIS. Dividing the given number of pints by 2, because there are $\frac{1}{2}$ as many quarts as pints, we obtain 26 qt. plus a remainder of 1 pt. We next divide 26 qt. by 4, because there are $\frac{1}{4}$ as many gallons as quarts, and we obtain 6 gal. and a remainder of 2 qt. This last quotient, with the several remainders annexed, forms the answer.

2. Reduce 4902 inches to rods.

OPERATION.

12)4902

$16\frac{1}{2}$)408 ft.+6 in.

2

33)816

24 rd.+$\frac{24}{2}$=12 ft.

Ans. 24 rd. 12 ft. 6 in.

ANALYSIS We divide successively by the numbers in the ascending scale in the same manner as in the preceding example. But in dividing the 408 ft. by $16\frac{1}{2}$, we first reduce 408 ft. to *halves* by multiplying by 2, and we have 816 *halves*; and reducing $16\frac{1}{2}$ to *halves*, we have 33 *halves*. Then dividing 816 by 33 we obtain 24 rd. plus a remainder of 24 *halves*=to 12 ft. which, with the preceeding remainder annexed to the last quotient, gives the answer.

RULE. I. *Divide the given number by that number of the scale which will reduce it to the next higher denomination.*

II. *Divide the quotient by the next higher number in the scale; and so proceed to the highest denomination required. The last quotient, with the several remainders annexed in a reversed order, will be the answer.*

EXAMPLES FOR PRACTICE.

3. How many pounds in 3460 ounces?

Ans. 216 lb. 4 oz.

4. How many shillings in 556 farthings?

Ans. 11s. 7d.

5. How many yards in 1242 inches?

6. How many gallons in 2347 pints?

7. Reduce 23547 troy grains to pounds.

Ans. 4 lb. 1 oz. 1 pwt. 3 gr.

8. Reduce 1597 quarts to bushels.

Ans. 49 bu. 3 pk. 5 qt.

9. Reduce 107520 oz. avoirdupois to pounds.

10. In 28635 sec. how many hours?

Ans. 7 hr. 57 min. 15 sec.

11. In 10000″ how many degrees?

Ans. 2° 46′ 40″.

12. In 11521 gr. apothecaries weight how many pounds?

Ans. 2lb 1 gr.

13. In 3561829 seconds how many weeks?

14. Reduce 67893 cu. ft. to cords.

15. In 1491 pounds how many hundred weight?

16. In 12244 pints how many hogsheads?

17. In 25600 sq. rd. how many acres? *Ans.* 160 A.

18. How many miles in 51200 rd.? *Ans.* 160 mi.

19. How many barrels in 6048 gills? *Ans.* 6 bbl.

20. In 316800 inches how many miles? *Ans.* 5 mi.

21. In 1728 how many gross? *Ans.* 12 gross.

22. In 4060 how many score? *Ans.* 203 score.

23. Reduce 1435 feet to fathoms.

24. Reduce 10000 sheets of paper to reams.

Ans. 20 reams 16 quires 16 sheets.

25. Reduce 27878400 sq. ft. to square miles.

PROMISCUOUS EXAMPLES IN REDUCTION.

1. Reduce 4 dollars 67 cents to cents. *Ans.* 467 cents.
2. Reduce 3724 mills to dollars. *Ans.* $3.724.
3. Reduce 9690 cents to dollars. *Ans.* $96.90.
4. Reduce 8 dollars to mills. *Ans.* 8000 mills.
5. In 91751 farthings how many pounds?
Ans. £95 11s. 5d. 3 far.
6. In 3 lb. 4 oz. 7 pwt. how many grains?
7. In 3 tons of cheese how many pounds?
8. How much will 4 cheese cost, each weighing 36 pounds, at 9 cents a pound? *Ans.* $12.96.
9. How much would 2 lb. 8 oz. 12 pwt. of gold dust be worth, at 72 cents a pwt.? *Ans.* $469.44.
10. Bought 1 T. 15 cwt. 36 lb. of sugar at 7 cents a pound; how much did it cost? *Ans.* $247.52.
11. Paid $23.50 for one hogshead of molasses, and sold it all at 50 cents a gallon; how much was the whole gain?
12. How many pounds in 6 barrels of flour?
13. How many bushels of oats in a load weighing 1280 pounds? *Ans.* 40 bu.
14. How many bushels of wheat in a load weighing 2175 pounds? *Ans.* 36 bu. 15 lb.
15. A grocer bought 3 barrels of flour at $6 a barrel, and sold it out at 4 cents a pound; how much did he gain on the whole? *Ans.* $5.52.
16. In a board 12 feet long and 2 feet wide, how many square feet? *Ans.* 24 sq. ft.
17. In a block of marble 6 feet long and 3 feet square, how my cubic feet? *Ans.* 54 cu. feet.
18. In a pile of wood 26 feet long 6 feet high and 3 feet wide, how many cubic feet? how many cords?
Ans. 468 cu. ft.; or 3 Cd. 84 cu. ft.

19. In 259200 cubic inches of hewn timber how many tons? *Ans.* 3 T.

20. How many square rods in a field 90 rods long and 75 rods wide? How many acres? *Ans.* 42 A. 30 sq. rd.

21. A pond of water measures 3 fathoms 2 feet 9 inches in depth; how many inches deep is it? *Ans.* 249 in.

22. What will 3 miles of telegraph cable cost at 12 cents a foot? *Ans.* $1900.80.

23. What is the age of a man 3 score and 5 years old? *Ans.* 65 years.

24. How much will I receive for a load of wheat weighing 2760 pounds at $1.50 per bushel? *Ans.* $69.

25. How many cubic feet in a stick of timber 32 feet long 2 feet wide and 1 foot thick? *Ans.* 64 cu. ft.

26. How many square feet in one acre?

27. In 176 yards how many rods? *Ans.* 32 rd.

28. A pile of wood is 16 feet long, 8 feet high, and 8 feet wide; how much is it worth at $3.50 a cord? *Ans.* $28.

29. What would be the value of a city lot 40 feet wide and 120 feet long, at 2 cents a square foot? *Ans.* $96.

30. A grocer bought 4 barrels of cider, at $2 a barrel, and after converting it into vinegar, he retailed it at 15 cents a gallon; how much was his whole gain. *Ans.* $10.90.

31. At 6 cents a pint how much molasses can be bought for $4.26? *Ans.* 8 gal. 3 qt. 1 pt.

32. An innkeeper bought a load of 40 bushels of oats, at 36 cents a bushel, and retailed them at 25 cents a peck; how much did he make on the load? *Ans.* $25.60.

33. What will be the cost of a hogshead of wine at 8 cents a gill? *Ans.* $161.28.

34. In 120 gross how many score? *Ans.* 864 score.

35. If a man walk 4 miles an hour, and 10 hours a day, how many miles can he walk in 24 days? *Ans.* 960 mi.

26. What will be the cost of 2 bu. 1 pk. 6 qt. of timothy seed, at 10 cents a quart? *Ans.* $7.80.

37. What would be the value of a silver goblet, weighing 8 oz. 14 pwt., at $.15 a pwt.? *Ans.* $26.10.

38. What will 16 reams of paper cost at 20 cents a quire? *Ans.* $64.

39. If 1 bushel of wheat make 45 pounds of flour, how many pounds will 500 bushels make? How many barrels? *Ans.* 114 bbl. 156 pounds.

40. Bought a gold chain, weighing 2 oz. 18 pwt. at $.90 a pwt.; how much did it cost? *Ans.* $52.20.

41. How many minutes more are there in the Summer than in the Autumn months? *Ans.* 1440 min.

42. How much will it cost to dig a cellar 24 ft. long, 18 ft. wide and 6 feet deep, at 1 cent a cubic foot? *Ans.* $25.92.

43. How many boxes, each containing 12 pounds, can be filled from a hogshead of sugar containing 9 cwt.? *Ans.* 75 boxes.

44. What will be the cost of 5 bales of cloth, each bale containing 15 pieces, and each piece measuring 26 yards, at $1.75 a yard?

45. If a cannon ball goes at the rate of 10 miles a minute, how many miles would it go, at the same rate, in 2 hours? *Ans.* 1200 miles.

46. At 11 cents a pound what will be the cost of 3 cwt. 2 qr. 21 lb. of coffee? *Ans.* $40.81.

47. If a man earn $30 a month, how much will he earn in 5 years? *Ans.* $1800.

ADDITION.

170. Compound numbers are added, subtracted, multiplied, and divided by the same general methods as are employed in simple numbers. The only modification of the operations and rules is that required for borrowing, carrying, and reducing by a *varying*, instead of a *uniform scale*.

1. What is the sum of 36 bu. 2 pk. 6 qt. 1 pt., 25 bu. 1 pk. 4 qt., 18 bu. 3 pk. 7 qt. 1 pt., 9 bu. 0 pk. 2 qt. 1 pt.?

OPERATION.

	bu.	pk.	qt.	pt.
	36	2	6	1
	25	1	4	0
	18	3	7	1
	9	0	2	1
Ans.	90	0	4	1

ANALYSIS. Arranging the numbers in columns, placing units of the *same* denomination under each other, we first add the units in the right hand column, or lowest denomination, and find the amount to be 3 pints, which is equal to 1 qt. 1 pt. We write the 1 pt. under the column of pints, and add the 1 qt. to the column of quarts. We find the amount of the second column to be 20 qt. which is equal to 2 pk. 4 qt. Writing the 4 qt. under the column of quarts, we add the 2 pk. to the column of pecks. Adding the column of pecks in the same manner, we find the amount to be 8 pk. equal to 2 bu. Writing 0 pk. under the column of pecks, we add the 2 bu. to the column of bushels. Adding the last column, we find the amount to be 90 bu. which we write under the left hand denomination, as in simple numbers. Hence the following

RULE I. *Write the numbers so that those of the same unit value will stand in the same column.*

II. *Beginning at the right hand, add each denomination as in simple numbers, carrying to each succeeding denomination one for as many units as it takes of the denomination added, to make one of the next higher denomination.*

EXAMPLES FOR PRACTICE.

(2.)

£.	s.	d.	far.
47	10	9	1
25	6	4	3
36	18	0	2
12	00	10	0
8	7	3	1
Ans. 130	3	3	3

(3.)

lb.	℥.	ʒ.	℈.	gr.
10	10	4	1	12
	9	5	2	10
14	4	0	0	16
6	0	7	1	00
	6	3	2	15
32	7	5	2	13

(4)

hhd.	gal.	qt.	pt.
24	21	3	1
102	42	2	0
38	9	0	1
42	50	1	0
207	60	3	0

(5)

T.	cwt.	lb.	oz.	dr.
3	12	15	10	11
	16	20	7	9
5	9	6	0	12
	18	17	14	00
10	15	59	1	00

(6)

da.	h.	min.	sec.
27	14	40	36
106	20	14	25
16	12	50	45
52	16	39	18

(7)

lb.	oz.	pwt.	gr.
16	11	18	21
26	9	15	10
11	10	00	8
4	6	12	00

(8)

mi.	fur.	rd.	yd.	ft.	in.
2	5	25	4	1	10
1	3	30	1	2	7
4	0	16	5	0	4
10	6	8	2	2	11

(9)

P.	sq. yd.	sq. ft.
12	20	5
9	15	6
15	10	7
20	26	3

10. What is the sum of 2S. 12°, 40′, 25″; 5S. 9°, 27′, 38″; 16° 10′ 50″; 1S. 16°?

11. What is the sum of 44A. 2R. 24P., 10A. 0R. 20P., 25A. 1R. 6A. 36P.? *Ans.* 86A. 1R.

12. What is the sum of 25 rd. 12 ft. 5 in., 28 rd. 9 ft. 10 in., 18 rd. 10 ft., 12 rd. 14 ft. 9 in.?

Ans. 2 fur. 5 rd. 14 ft.

13. What is the sum of 5 Cd. 6 cd. ft. 9 cu. ft., 4 Cd. 3 cd ft. 12 cu. ft., 10 Cd. 14 cu. ft., 2 Cd. 7 cd. ft.?

Ans. 23 Cd. 2 cd. ft. 3 cu. ft.

14. What is the sum of 40 yd. 2 ft. 10 in., 37 yd. 1 ft. 9 in., 28 yd. 11 in., 10 yd. 2 ft., 15 yd.?

Ans. 132 yd. 1 ft. 6 in.

15. What is the sum of 13 Cd. 60 cu. ft. 164 cu. in., 25 Cd. 75 cu. ft., 18 Cd. 25 cu. ft. 540 cu. in., 8 Cd. 1030 cu. in.? *Ans.* 65 Cd. 33 cu. ft. 6 cu. in.

16. A grocer bought 4 hhd. of sugar; the first weighed 11 cwt. 2 qr. 21 lb.; the second 10 cwt. 1 qr. 16 lb.; the third 10 cwt. 22 lb.; and the fourth 9 cwt. 3 qr. How much did the whole weigh? *Ans.* 2T. 2 cwt. 9 lb.

17. A man has a farm divided into three fields; the first contains 26 A. 2 R. 30 P.; the second, 48 A. 27 P.; an the third, 35 A. 2 R. How many acres in the farm?

Ans. 110 A. 1 R. 17 P.

18. If a printer one day use 2 bundles 1 ream 10 quires of paper, the next day 3 bundles 1 ream 12 quires, 20 sheets, and the next, 4 bundles 9 quires, how much does he use in the three days?

Ans. 10 bundles 1 ream 11 quires 20 sheets.

19. A tailor used, in one year, 3 gross 6 doz. 10 buttons, another year, 2 gross 9 doz. 9 buttons, and another year, 4 gross 7 doz.; how many did he use in the three years?

SUBTRACTION.

171. From 24 lb. 6 oz. 5 pwt. 12 gr. take 14 lb. 9 oz. 10 pwt. 7 gr.

OPERATION.

	lb.	oz.	pwt.	gr.
	24	6	5	12
	14	9	10	7
Ans.	9	8	15	5

ANALYSIS. Writing the subtrahend under the minuend, placing units of the same denomination under each other, we subtract 7 gr. from 12 gr. and write the remainder, 5 gr., underneath. Since we cannot subtract 10 pwt. from 2 pwt., we add 1 oz. or 20 pwt. to the 5 pwt., and subtract 10 pwt. from the sum, 25 pwt, and write the remainder, 15 pwt., underneath. Having added 20 pwt. or 1 oz. to the minuend, we now add 1 oz. to the 9 oz. in the subtrahend, making 10 oz; but as we cannot take 10 oz. from 6 oz. we add 1 lb. or 12 oz. to the 6 oz. making 18 oz. and subtracting 10 oz. from 18 oz. we write the remainder, 8 oz. under the denomination of ounces. Having added 1 lb. to the minuend, we now add 1 lb. to the 14 lb. in the subtrahend, and subtracting 15 lb. from 24 lb. as in simple numbers, we write the remainder, 9 lb. under the denomination of pounds. Hence

RULE. I. *Write the subtrahend under the minuend, so that units of the same denomination shall stand under each other.*

II. *Beginning at the right hand, subtract each denomination separately, as in simple numbers.*

III. *If the number of any denomination in the subtrahend exceed that of the same denomination in the minuend, add to the number in the minuend as many units as make one of the next higher denomination, and then subtract; in this case add* 1 *to the next higher denomination of the subtrahend before subtracting. Proceed in the same manner with each denomination.*

EXAMPLES FOR PRACTICE.

(2)

	cwt.	qr.	lb.	oz.	dr.
From	18	1	14	9	8
Take	5	2	20	6	10
Rem.	12	2	19	2	14

(3)

	hhd.	gal.	qt.	pt.
	7	28	2	1
	3	42	3	0
	3	48	3	1

(4)

℔.	℥.	ʒ.	℈.	gr.
12	7	3	1	11
8	5	4	2	15

(5)

bu.	pk.	qt.	pt.
104	2	6	0
56	3	4	1

(6)

mi.	fur.	rd.	yd.	ft.	in.
40	5	30	3	2	10
14	6	15	4	1	01

(7)

A.	R.	P.
400	2	25
325	1	30

(8)

wk.	da.	hr.	min.	sec.
10	4	16	40	22
4	5	12	45	50

(9)

S.	°	′	″
6	25	45	38
4	28	40	50

(10)

T.	cwt.	qr.	lb.	oz.
14	5	2	18	9
10	14	3	12	14

(11)

Cd.	cd.ft.	cu.ft.	cu.in.
120	4	6	520
94	7	12	1500

(12)

yd.	ft.	in.
74	2	6
9	2	9

(13)

Cd.	cu.ft.
325	80
128	112

(14)

sq.yd.	sq.ft.	sq.in
27	6	96
14	8	120

15. From 125 mi. 6 fur. take 90 mi. 4 fur. 25 rd.

Ans. 35 mi. 1 fur. 15 rd.

16. A man bought 1 hhd. of molasses, and sold 42 gal. 3 qt. 1 pt.; how much remained? *Ans.* 20 gal. 1 pt.

17. A person bought 9 T. 14 cwt. 3 qr. of coal, and having burned 4 T. 15 cwt. sold the remainder; how much did he sell? *Ans.* 4 T. 19 cwt. 3 qr.

18. If from a tub of butter containing 1 cwt. 21 lb there be sold 24 lb. 8 oz. how much remains?

Ans. 96 lb. 8 oz.

19. From a pile of wood containing 42 Cd. 5 cd ft. there was sold 16 Cd. 6 cd. ft. 12 cu. ft.; how much remained?

Ans. 25 Cd. 6 cd. ft. 4 cu. ft.

20. If from a field containing 37 A. 3 R. 26 P. there be taken 14 A. 2 R. 30 P., how much will there be left?

21. A farmer having raised 50 bu. 2 pk. of wheat, kept for his own use 25 bu. 3 pk.; how much did he sell?

Ans. 24 bu. 3 pk.

22. The distance from New York to Albany is 150 miles; when a man has traveled 84 mi. 6 fur. 30 rd. of the distance, how much farther has he to travel?

Ans. 65 mi. 1 fur. 10 rd.

23. What is the difference in the longitude of two places one 71° 20′ 26″, and the other 44° 35′ 58″ West?

Ans. 26° 44′ 28″.

24. If from a hogshead of molasses 10 gal. 2 qt. be drawn at one time, 9 gal. 3 qt. at another, and 14 gal. at another, how much will remain? *Ans.* 28 gal. 3 qt.

85. From a section of land containing 640 acres, there was sold at one time 140 A. 2. R. 36 P., at another time 200 A. 1 R., and at another time 75 A. 28 P.. how much remained? *Ans.* 223 A. 3. R. 16 P.

MULTIPLICATION.

172. 1. A farmer has 8 fields, each containing 4 A. 2 R. 27 P.; how much land in all?

OPERATION.

A.	R.	P.
4	2	27
		8
37	1	16

ANALYSIS. In 8 fields are 8 times as much land as in 1 field. We write the multiplier under the lowest denomination of the multiplicand, and proceed thus; 8 times 27 P. are 216 P., equal to 5 R. 16 P.; and we write the 16 P. under the number multiplied. Then, 8 times 2 R. are 16 R., and 5 R. added make 21 R., equal to 5 A. 1 R.; and we write the 1 R. under the number multiplied. Again, 8 times 4 A. are 32 A. and 5 A. added make 37 A., which we write under the same denomination in the multiplicand, and the work is done. Hence

RULE. I. *Write the multiplier under the lowest denomination of the multiplicand.*

II. *Multiply as in simple numbers, and carry as in addition of compound numbers.*

EXAMPLES FOR PRACTICE.

(2.)

	hhd.	gal.	qt.	pt.
	6	20	2	1
				3
Ans.	18	61	3	1

(3.)

	bu.	pk.	qt.	pt.
	9	2	6	1
				4
	38	3	2	0

(4.)

	lb.	oz.	pwt.	gr.
	12	8	14	16
				5
	63	7	13	8

(5.)

	T.	cwt.	lb.	oz.
	10	15	20	8
				6
	64	14	23	0

6. Multiply 14 A. 2 R. 26 P. by 8.

Ans. 117 A. 1 R. 8 P.

7. Multiply 6 yd. 2 ft. 9 in. by 12. *Ans.* 83 yd.

8. Multiply 7 ℔. 8 ℥. 5 ʒ. 1 ℈. 10 gr. by 7.

Ans. 54 ℔. 0 ℥. 6 ʒ. 1 ℈. 10 gr.

9. Multiply 24 bu. 1 pk. 6 qt. by 10.

10. Multiply 9 cu. yd. 15 cu. ft. 520 cu. in. by 5.

Ans. 47 cu. yd. 22 cu. ft. 872 cu. in.

11. Multiply £84 12s. 6d. 2 far. by 9.

12. If a pipe discharge 4 hhd. 20 gal. 3 qt. of water in 1 hour, how much will it discharge in 5 hours, at the same rate? *Ans.* 21 hhd. 40 gal. 3 qt.

13. If a load of coal by the long ton weigh 1 T. 4 cwt. 2 qr. 20 lb. what will be the weight of 6 loads?

Ans. 7 T. 8 cwt. 8 lb.

14. If 1 acre of land produce 26 bu. 3 pk. 4 qt. of wheat, how much will 11 acres produce?

15. If a man travel 30 mi. 4 fur. 20 rd. in 1 day, how far will he travel in 9 days, at the same rate?

16. What is the weight of 3 dozen silver spoons, each dozen weighing 2 lb. 10 oz. 12 pwt. 14 gr.?

Ans. 8 lb. 7 oz. 17 pwt. 18 gr.

17. If a wood chopper can cut 2 cd. 6 cd. ft. 8 cu. ft. of wood in a day, how many cords can he cut in 10 days?

18. In 20 barrels of potatoes, each containing 2 bu. 3 pk. 6 qt., how many bushels? *Ans.* 58 bu. 3 pk.

19. A grocer bought 14 barrels of sugar, each weighing 5 cwt. 1 qr. 15 lb.; how much did the whole weigh?

20. If the sun, on an average, change his longitude 59′ 9″ each day, how much will be the change in 25 days?

21. If 1 qt. 1 pt. 3 gi. of wine fill 1 bottle, how much will be required to fill 3 dozen bottles of the same capacity?

22. If a yard of cloth cost £2 10s. 9d. how much will 18 yards cost? *Ans.* £45 13s. 6d.

23. If a person average 8 hr. 20 min. 40 sec. of sleep daily, how much will he sleep in 30 days?

Ans. 10 da. 10 hr. 20 min.

24. How many cords of wood in 8 piles, each containin 40 cd. ft. 104 cu. ft. 432 cu. in.?

Ans. 46 Cd. 4 cd. ft. 2 cu. ft.

25. If each silver table-spoon weigh 1 oz. 12 pwt. 16 gr., what is the weight of 1 dozen spoons?

26. If the moon's average daily motion is 33° 10′ 35″, how much of her orbit does she traverse in 21 days?

27. How much land in 12 lots, each containing 2 A. 120 P.? *Ans.* 33 A.

28. How many bushels of wheat in 48 sacks, each containing 165 pounds? *Ans.* 132 bu.

29. If a locomotive move 4 fur. 36 rd. in one minute, how far will it move in one hour? *Ans.* 36 mi. 6 fur.

30. If a family consume 2 gal. 1 qt. 1 pt. of molasses in 1 week, how much will they consume in 1 year?

Ans. 1 hhd. 60 gal. 2 qt.

31. If it take a man 5 hr. 42 min. 50 sec. to saw cord of wood, how long will it take him to saw 16 cords?

Ans. 91 hr. 25 min. 20 sec.

32. How many bushels of apples can be put into 75 barrels, each barrel containing 3. bu. 1 pk.?

Ans. 243 bu. 3 pk.

33. If a man can build 3 rd. 12 ft. 10 in. of wall in 1 day, how much can he build in 10 days?

Ans. 37 rd. 12½ ft. 4 in.

34. If a man can mow 2 A. 96 P. of grass in a day, how much can 27 men mow, at the same rate?

DIVISION.

173. If 4 acres of land produce 102 bu. 2 pk. 2 qt. of wheat, how much will 1 acre produce?

OPERATION.

	bu.	pk.	qt.	pts.
4)102	3	2		
25	2	6	1	

ANALYSIS. One acre will produce ¼ as much as 4 acres. Writing the divisor on the left of the dividend, we divide 102 bu. by 4, and we obtain a quotient of 25 bu., and a remainder of 2 bu. We write the 25 bu. under the denomination of bushels, and reduce the 2 bu. to pecks, making 8 pk., and the 3 pk. of the dividend added makes 11 pk. Dividing 11 pk. by 4, we obtain a quotient of 2 pk. and a remainder of 3 pk.; writing the 2 pk. under the order of pecks, we next reduce 3 pk. to quarts, adding the 2 qt. of the dividend, making 26 qt., which divided by 4 gives a quotient of 6 qt. and a remainder of 2 qt. Writing the 6 qt. under the order of quarts, and reducing the remainder, 2 qt., to pints, we have 4 pt., which divided by 4 gives a quotient of 1 pt., which we write under the order of pints, and the work is done.

2. A farmer put 132 bu. 1 pk. of apples into 46 barrels; how many bu. did he put into a barrel?

OPERATION.

	bu.	pk.	
46)132		1	(2 bu.
	92		
	40		
	4		
161			(3 pk.
	138		
	23		
	8		
184			(4 qt.
	184		

Ans. 2 bu. 3 pk. 4 qt.

When the divisor is large we divide by long division, as shown in the operation. From these examples we derive the following

RULE. I. *Divide the highest denomination as in simple numbers, and each succeeding denomination in the same manner, if there be no remainder.*

II. *If there be a remainder after dividing any denomination, reduce it to the next lower denomination, adding in the given number of that denomination, if any, and divide as before.*

III. *The several partial quotients will be the quotient required.*

EXAMPLES FOR PRACTICE.

(3)

	A.	R.	P.
2)95	2	30	
47	3	15	

(4)

	lb.	oz.	pwt.	gr.
3)52	4	16	18	
17	5	12	6	

(5)

	wk.	da.	h.	min.	sec.
7)33	5	23	45	10	
4	5	20	32	10	

(6)

	bu.	pk.	qt.
6)88	3	4	
14	3	2	

(7)

	lb.	ʒ.	ʒ.	℈.	gr.
5)28	9	4	2	5	
5	9	0	2	17	

(8)

	gal.	qt.	pt.
9)376	3	1	
41	3	1	

(9)

	hhd.	gal.	qt.	pt.
12)9	28	2	0	
0	49	2	1	

(10)

	A.	R.	P.
9)129	2	25	
14	1	25	

(11)

	mi.	fur.	rd.	ft.	in.
7)217	5	19	12	6	
31	0	31	6	6	

(12)

	lb.	oz.	pwt.	gr
11)185	1	19	13	
16	9	19	23	

13. Divide £185. 17s. 6d. by 8.

Ans. £23. 4s. 8d. 1 far.

14. Divide 16 lb. 13 oz. 10 dr. by 6.

Ans. 2 lb. 12 oz. 15 dr.

15. Divide 358 A. 1 R. 17 P. 6 sq. yd. 2 sq. ft. by 7.

Ans. 51 A. 31 P. 8 sq. ft.

16. Divide 192 bu. 3 pk. 1 qt. 1 pt. by 9.

Ans. 21 bu. 1 pk. 5 qt. 1 pt.

17. Divide 9 hhd. 28 gal. 2 qt. by 12.

Ans. 49 gal. 2 qt. 1 pt.

18. Divide 328 yd. 1 ft. 3 in. by 21.

Ans. 15 yd. 1 ft. 11 in.

19. Divide 36℔. 11 ℥. 4 ʒ. 2 ℈. 7 gr. by 11.

Ans. 3℔. 4 ℥. 2 ʒ. 1 ℈. 17 gr.

20. Divide 16 cwt. 3 qr. 18 lb., long ton weight, by 32.

Ans. 2 qr. 3 lb. 3 oz.

21. If a steamboat run 174 mi. 26 rd. in 14 hours, how far does she run in 1 hour?

22. A farm containing 322 A. 2 R. 10 P. is to be divided equally among 13 persons; how much will each have?

Ans. 24 A. 3 R. 10 P.

23. A cartman drew 38 cd. 5 cd. ft. 6 cu. ft. of wood, at 30 loads; how much did he average per load?

Ans. 1 cd. 2 cd. ft. 5 cu. ft.

24. If 24 barrels of flour cost £98. 16s., how much will 1 barrel cost? *Ans.* £4. 2s. 4d.

25. If a vessel sail 153° 16′ 12″ in 27 days, how far does she sail on an average per day?

Ans. 5° 40′ 36″.

26. If 3 dozen spoons weigh 9 lb. 8 oz. 12 gr., how much does each spoon weigh? *Ans.* 3 oz. 4 pwt. 11 gr.

PROMISCUOUS EXAMPLES.

1. A farmer raised 200 bu. 2 pk. of barley, 175 bu. 3 pk. of corn, 320 bu. 1 pk. of oats, and 225 bu. 2 pk. of rye; what was the whole quantity of grain raised?

2. A person having bought 325 A. 2 R. of land, sold 150 A. 1 R. 25 P. of it; how much had he remaining?

3. What is the whole weight of 72 hogsheads of sugar, each weighing 12 cwt. 3 qr.? *Ans.* 45 T. 18 cwt.

4. If a railroad car run 148 miles 4 fur. in 8 hours, what is the average rate of speed per hour?

5. A grocer having purchased 98 cwt. 2 qr. of sugar, sold 10 cwt. 1 qr. 20 lb. to one man, and 18 cwt. 16 lb. to another; how much remained unsold?

6. Bought 12 tea-spoons, each weighing 16 pwt. 20 gr., and 6 table-spoons, each weighing 1 oz. 12 pwt.; what was their total weight? *Ans.* 1 lb. 7 oz. 14 pwt.

7. A farmer raised 24 T. 17 cwt. of hay; he sold 5 loads, each weighing 1 T. 8 cwt. 21 lb.; how much has he remaining? *Ans.* 17 T. 15 cwt. 95 lb.

8. A jeweler having 36 lb. 10 oz. 14 pwt. of silver, uses 21 lb. 6 oz. of it, and then manufactures the remainder into 8 tea-pots; what is the weight of each?

Ans. 1 lb. 11 oz. 1 pwt. 18 gr.

9. A man purchasing 2 A. 140 sq. rd. of land, reserves $\frac{1}{2}$ an acre for his own use, and divides the remainder in 4 equal lots; how much does each lot contain?

Ans. 95 sq. rd.

10. How many pounds of sugar in 28 barrels, each containing 3 cwt. 1 qr. 17 lb.? *Ans.* 9576 pounds.

11. If from a piece of land containing 5 A. 3 R., 2 A 72 P. be taken, how many square rods will remain?

12 Divide a tract of land containing 1299500 square rods into 25 farms of equal area; how many acres will there be in each?

Ans. 324 A. 3 R. 20 P.

13. A merchant buys 3 hogsheads of molasses at 30 cents a gallon, and sells it at 45 cents; how much does he gain on the whole?

14. What is the cost of 3 chests of tea, each weighing 2 cwt. 2 qr. 18 lb., at $.84 a pound? *Ans.* 225.12.

15. How many steps of 30 inches each must a person take in walking 12 miles?

16. If a man buy 10 bushels of chestnuts, at $3 a bushel, and sell them at 10 cents a pint, how much is his whole gain? *Ans.* $34.

17. How many times will a wheel 13 ft. 4 inches in circumference turn round in going 12 miles?

Ans. 4752.

18. If 8 horses eat 12 bu. 3 pk. of oats in 3 days, how many bushels will 20 horses eat in the same time?

Ans. 31 bu. 3 pk. 4 qt.

19. How much sugar at 9 cents a pound must be given for 2 cwt. 43 lb. of pork at 6 cents a pound?

Ans. 162 pounds.

20. How many cubic feet in a room 18 feet long, 16 feet wide, and 10 feet high?

21. A person wishes to ship 720 bushels of potatoes in barrels, which shall hold 3 bu. 3 pk. each, how many barrels must he use? *Ans.* 192.

22. How many rods of fence will inclose a farm a mile square? *Ans.* 1280 rods.

23. If granite weigh 175 pounds a cubic foot, what is the weight of a cubic yard? *Ans.* 2 T. 7 cwt. 25 lb.

CANCELLATION.

174. **Cancellation** is the process of rejecting equal factors from numbers sustaining to each other the relation of dividend and divisor.

It has been shown (**70**) that the dividend is equal to the product of the divisor multiplied by the quotient. Hence, if the dividend can be resolved into two factors, one of which is the divisor, the other factor will be the quotient.

1. Divide 72 by 9.

OPERATION.

Divisor. $9)9\times8$ Dividend.

8 Quotient.

ANALYSIS. We see in this example, that 72 is composed of the factors 9 and 8, and that the factor 9, is equal to the divisor. Therefore we reject the factor 9, and the remaining factor, 8, is the quotient.

174. Whenever the dividend and divisor are each composite numbers, the factors common to both may first be rejected without altering the final result.

2. What is the quotient of 48 divided by 24?

OPERATION.

$$\frac{48}{24}=\frac{3\times\cancel{8}\times2}{3\times\cancel{8}}=2\ Ans.$$

ANALYSIS. We first *indicate* the operation to be performed, by writing the dividend above a line, and the divisor below it. We resolve 48, into the factors 3, 8 and 2, and 24 into the factors 3, and 8. We next cancel the factors 3, and 8, which are common to the dividend and divisor, and we have left the factor 2, in the dividend, which is the quotient.

NOTE. When all the factors or numbers in the dividend are cancelled, 1 should be retained.

175. If any two numbers, one in the dividend and one in the divisor, contain a common factor, we may reject that factor.

3. In 15 times 63, how many times 45?

OPERATION.

$$\frac{\overset{3}{\cancel{15}}\times\overset{7}{\cancel{63}}}{\cancel{45}}=21 \text{ Ans.}$$

$\cancel{9}$

ANALYSIS. In this example we see that 5 will divide 15 and 45; so we reject 5 as a factor of 15, and retain the factor 3, and also as a factor of 45, and retain the factor 9. Again 9 will divide 9 in the divisor, and 63 in the dividend. Dividing both numbers by 9, 1 will be retained in the divisor, and 7 in the dividend. Finally the product of $3\times 7=21$, the quotient.

4. What is the quotient of $25\times18\times6\times4$, divided by $15\times4\times9\times3$?

OPERATION.

$$\frac{\overset{5}{\cancel{25}}\times\overset{2}{\cancel{18}}\times\overset{2}{\cancel{6}}\times\cancel{4}}{\overset{3}{\cancel{15}}\times\cancel{4}\times\cancel{9}\times\cancel{3}}=\frac{5\times2\times2}{3}=\frac{20}{3}=6\tfrac{2}{3} \text{ Ans.}$$

ANALYSIS. In this, as in the preceeding example, we reject all the factors that are common to both dividend and divisor, and we have remaining the factor 3 in the divisor, and the factors 5, 2, and 2 in the dividend. Completing the work, we have $\frac{20}{3}=6\frac{2}{3}$, *Ans.*

From the preceeding examples and illustrations we derive the following

RULE.—I. *Write the numbers composing the dividend above a horizontal line, and the numbers composing the divisor below it.*

II. *Cancel all the factors common to both dividend and divisor.*

III. *Divide the product of the remaining factors of the dividend by the product of the remaining factors of the divisor, and the result will be the quotient.*

NOTES. 1. Rejecting a factor from any number is dividing the number by that factor.

2. When a factor is cancelled, the unit, 1, is supposed to take its place.

3. *One* factor in the dividend will cancel only *one equal* factor in the divisor.

4. If all the factors or numbers of the divisor are cancelled, the product of the remaining factors of the dividend will be the quotient.

5. By many it is thought more convenient to write the factors of the dividend on the right of a *vertical* line, and the factors of the divisor on the left.

EXAMPLES FOR PRACTICE.

1. Divide the product of $12\times8\times6$ by $8\times4\times3$.

FIRST OPERATION.

$$\frac{\overset{3}{\cancel{12}}\times\cancel{8}\times\cancel{6}^{2}}{\cancel{8}\times\cancel{4}\times\cancel{3}}=\frac{3\times2}{1}=6\ \textit{Ans.}$$

SECOND OPERATION.

$\cancel{8}$	$\cancel{12}^{3}$
$\cancel{4}$	$\cancel{8}$
$\cancel{3}$	$\cancel{6}^{2}$

6 *Ans.*

2. Divide the product of $25\times18\times4\times3$, by $7\times6\times5\times3$.

FIRST OPERATION.

$$\frac{\overset{5}{\cancel{25}}\times\overset{3}{\cancel{18}}\times4\times\cancel{3}}{7\times\cancel{6}\times\cancel{5}\times\cancel{3}}=\frac{5\times3\times4}{7}=\frac{60}{7}=8\tfrac{4}{7}\ \textit{Ans.}$$

7	$\cancel{25}^{5}$
$\cancel{6}$	$\cancel{18}^{3}$
$\cancel{5}$	4
$\cancel{3}$	$\cancel{3}$
7	60

$8\frac{4}{7}$ *Ans.*

3. Divide the product of $36\times10\times7$ by $14\times5\times9$.

Ans. 4.

4. What is the quotient of $21\times8\times40\times3$ divided by $12\times7\times20$?

Ans. 12.

5. What is the quotient of $64\times18\times9$ divided by $30\times27\times4$?

Ans. $3\frac{1}{5}$.

6. Divide the product of $120\times44\times6$ by $60\times11\times8$.

Ans. 6

7. Multiply 200 by 60, and divide the product by 50 multiplied by 48. *Ans.* 5.

8. Multiply 8 times 32 by 6 times 27, and divide the product by 9 times 96. *Ans.* 48.

9. What is the quotient of $21\times8\times60\times8\times6$ divided by $7\times12\times3\times8\times3$? *Ans.* 80.

10. What is the quotient of $18\times6\times4\times42$ divided by $4\times9\times3\times7\times6$? *Ans.* 4.

11. If $18\times5\times9\times66$ be divided by $40\times22\times6$, what is the quotient? *Ans.* $10\frac{1}{8}$.

12. The product of the numbers 26, 11, and 21, is to be divided by the product of the numbers 14 and 13; what is the quotient? *Ans.* 33.

13. The product of the numbers 48, 72, 28 and 5, is to be divided by the product of the numbers 84, 15, 7 and 6; what is the quotient? *Ans.* $9\frac{1}{7}$.

14. How many tons of hay at \$9 a ton, must be given for 27 cords of wood, at \$4 a cord? *Ans.* 12 tons.

15. How many bushels of corn, worth 60 cents a bushel, must be given for 25 bushels of rye, worth 90 cents a bushel? *Ans.* $37\frac{1}{2}$ bushels.

16. How many peaches worth 2 cents each must be given for 48 oranges, at 3 cents each? *Ans.* 72

17. How many days work, at 75 cents a day, will pay for 30 pounds of coffee, at 15 cents a pound? *Ans.* 6 days.

18. How many suits of clothes, at \$18 a suit, can be made from 5 pieces of cloth, each piece containing 24 yards, at \$3 a yard? *Ans.* 20 suits.

19. How many tubs of butter, each containing 48 pounds, at 14 cents a pound, must be given for 3 boxes of tea, each containing 42 pounds, worth 60 cents a pound?

Ans. $11\frac{1}{4}$.

20. How many days work, at 84 cents a day, will pay for 36 bushels of corn worth 56 cents a bushel?

Ans. 24.

21. A farmer exchanged 45 bushels of potatoes worth 30 cents a bushel, for 15 pounds of tea; what was the tea worth a pound? *Ans.* 90 cents.

22. A grocer bought 120 pounds of cheese, at 9 cents a pound, and paid in molasses, at 45 cents a gallon; how many gallons of molasses paid for the cheese?

Ans. 24 gallons.

23. Gave 12 barrels of flour, at $7 a barrel, for hay worth $18 a ton; how many tons of hay was the flour worth? *Ans.* $4\frac{2}{3}$ tons.

24. Sold 8 firkins of butter, each weighing 56 pounds, at 15 cents a pound, and received in payment 3 boxes of tea, each containing 40 pounds; how much was the tea worth a pound? *Ans.* 56 cents.

25. A man took 6 loads of apples to market, each load containing 14 barrels, and each barrel 3 bushels. He sold them at 50 cents a bushel, and received in payment 9 barrels of sugar, each weighing 210 pounds; how much was the sugar worth a pound? *Ans.* $6\frac{2}{3}$ cents.

26. A grocer sold 12 boxes of soap, each containing 51 pounds, at 10 cents a pound; he received in payment a certain number of barrels of potatoes, each containing 3 bushels, at 30 cents a bushel; how many barrels did he receive? *Ans.* 68 barrels.

27. A man sold 4 loads of barley, each load containing 60 bushels, at 70 cents a bushel, and received in payment 2 pieces of cloth, each piece containing 35 yards, how much was the cloth worth a yard? *Ans.* $2.40.

ANALYSIS.

176. **Analysis**, in arithmetic, is the process of solving problems independently of set rules, by tracing the relations of the given numbers and the reasons of the separate steps of the operation according to the special conditions of each question.

177. In solving questions by analysis, we generally reason from the *given number* to *unity*, or 1, and then from unity, or 1, to the *required number*.

178. United States money is reckoned in dollars, dimes, cents, and mills, one dollar being uniformly valued in all the States at 100 cents; but in most of the States money is sometimes still reckoned in pounds, shillings and pence.

NOTE. At the time of the adoption of our decimal currency by Congress, in 1786, the *colonial currency*, or *bills of credit*, issued by the colonies, had depreciated in value and this depreciation, being unequal in the different colonies, gave rise to the different values of the State currencies; and this variation continues wherever the denomination of shillings and pence are in use.

Georgia Currency.

Georgia, South Carolina,.....$1=4s. 8d.=56d.

Canada Currency.

Canada, Nova Scotia,.....$1=5s.=60d.

New England Currency.

New England States, Indiana, Illinois, Missouri, Virginia, Kentucky, Tennessee, Mississippi, Texas,.....$1=6s.=72d.

Pennsylvania Currency.

New Jersey, Pennsylvania, Delaware, Maryland,.....$1=7s. 6d.=90d.

New York Currency.

New York, Ohio, Michigan, North Carolina,.....$1=8s.=96d.

In many of the States it is customary to give the retail price of articles in shillings and pence, and the cost of the whole in dollars and cents.

The following will be found an easy, short, and practical method of reducing currencies to dollars and cents.

EXAMPLES FOR PRACTICE.

1. What will be the cost of 36 bushels of apples, at 3 shillings a bushel, New England Currency?

OPERATION.

$$\begin{array}{l|l} 36\times3=108\text{s.} & \cancel{36}^{6} \\ 108\div6=\$18 \quad \text{Or } \cancel{6} & 3 \\ \hline & \$18, \textit{ Ans.} \end{array}$$

ANALYSIS.—Since 1 bushel costs 3 shillings, 36 bushels will cost 36 times 3s., or $36\times3=108$s.; and as 6s. make 1 dollar, New England currency, there are as many dollars in 108s. as 6 is contained times in 108, or $108\div6=\$18$.

2. What will 112 bushels of barley cost, at 5s. 6d. per bushel, New York currency?

OPERATION.

$$\begin{array}{l|l} & \cancel{112}^{7} \\ \cancel{96} & \cancel{66}^{11} \\ \hline & \$77 \end{array} \quad \text{Or} \quad \begin{array}{l|l} & \cancel{112}^{7} \\ \cancel{2} & 11 \\ \cancel{8} & \\ \hline & \$77 \textit{ Ans.} \end{array}$$

ANALYSIS.—We multiply the number of bushels by the price, and divide the result by the value of 1 dollar as in the first example, reducing both the *price* and 1 *dollar* to *pence*, and we obtain \$77. Or, when the price is an aliquot part of a shilling, the price may be reduced to an improper fraction for a multiplier, thus; 5s. 6d$=5\frac{1}{2}$s.$=\frac{11}{2}$s., the multiplier. The value of a dollar being 8s., we divide by 8 as in the operation. Hence To find the cost of articles in dollars and cents, when the price is in shillings and pence,

Multiply the commodity by the price, and divide the product by the value of one dollar in the required currency, reduced to the same denominational unit as the price.

3. What will 180 cords of wood cost at 8s. 4d. per cord, Pennsylvania currency?

OPERATION.

$$\begin{array}{r|l} & \overset{2}{\cancel{180}} \\ \cancel{90} & 100 \\ \hline & \$200 \end{array} \qquad \text{Or,} \qquad \begin{array}{r|l} & \overset{4}{\cancel{180}} \\ & 25 \\ \cancel{3} & 2 \\ \cancel{25} & \\ \hline & \$200, \textit{ Ans.} \end{array}$$

ANALYSIS.—Multiply the quantity by the price in pence, and divide the product by the value of 1 dollar in pence; or, reduce the shillings and pence, both of the price and of the dollar, to the fraction of a shilling before multiplying and dividing, thus; 8s. 4d.$=8\frac{1}{3}$s. $=\frac{25}{3}$s., the multiplier. The value of the dollar being 7s. 6d. $=7\frac{1}{2}$s. $=\frac{15}{2}$s. we divide by $\frac{15}{2}$, as in the operation.

4. What will be the cost of $7\frac{1}{2}$ yards of cloth, at 6s. 8d. New York currency?

OPERATION.

$$\begin{array}{r|l} 2 & \overset{5}{\cancel{15}}_{5} \\ \cancel{3} & \cancel{20} \\ {}^{2}\cancel{8} & \\ \hline 4 & 25.00 \\ \hline & \$6.25 \quad \textit{Ans.} \end{array}$$

ANALYSIS.—We reduce the quantity and the price to improper fractions, before multiplying.

NOTE.—When there is a remainder in the dividend, it may be reduced to cents and mills by annexing two or three ciphers and continuing the division.

5. What will 7 hhd. of molasses cost at 1s. 3d. per quart, Georgia currency?

OPERATION.

$$\begin{array}{r|l} & \cancel{7} \\ & 63 \\ & \cancel{4} \\ \overset{2}{\cancel{56}} & 15 \\ \hline 2 & 945.00 \\ \hline & \$472.50 \quad \textit{Ans.} \end{array}$$

ANALYSIS.—In this example we first reduce 7 hhd. to quarts, by multiplying by 63, and 4, and then multiply by the price, either reduced to pence or to an improper fraction, and divide by the value of 1 dollar reduced to the same denomination as the price.

6. Sold 8 firkins of butter, each containing 56 pounds at 1s. 3d. per pound, and received in payment tea at 6s. 8d. per pound; how many pounds of tea would pay for the butter?

OPERATION.

2	$ 28
10	$6 3
$0	15

Ans. 84 pounds.

ANALYSIS.—The operation in this is similar to the preceding examples, except that we divide the cost of the butter by the price of *a unit* of the article received in payment, reduced to the same denominational unit as the price of *a unit* of the article sold. The result will be the same in whatever currency.

7. What will be the cost of a load of oats containing 64 bushels at 2s. 6d. a bushel, New York currency?

Ans. $20.

8. At 9d. a pound, what will be the cost of 120 pounds of sugar, New England currency? *Ans.* $15.

9. What will be the value of a load of potatoes, measuring 35 bushels, at 2s. 3d. a bushel, Penn. currency?

Ans. $10.50.

10. What will be the cost of 240 bushels of wheat, at 9s. 4d. a bushel, Michigan currency? *Ans.* $280.

11. In New Jersey currency? *Ans.* $298.66⅔.

12. In Illinois currency? *Ans.* $373.33⅓.

13. In South Carolina currency? *Ans.* $480.

14. In Virginia currency? *Ans.*

15. In Ohio currency? *Ans.*

16. In Canada currency? *Ans.* $448.

17. How many days work at 7s. 6d. a day, must be given for 5 bushels of wheat at 10s. a bushel? *Ans.* 6⅔ days.

18. What will be the cost of 5 casks of rice, each weighing 168 pounds, at 3d. per pound, South Carolina currency?

Ans. $45.

19. How many pounds of sugar, at 9d. per pound, must be given for 18 bushels of apples, at 2s. 7d. per bushel?

Ans. 62 pounds.

20. Bought 3 casks of catawba wine, each cask containing 64 gallons, at 7s. 9d. per quart, Ohio currency; what was the cost of the whole? *Ans.* $744.

21. What will it cost to build 150 rods of wall, at 3s. 8d. per rod, Canada currency? *Ans.* $110.

22. How many pounds of butter, at 18d. a pound, must be given for 12 pounds of tea, at 5s. 4d. a pound?

Ans. $42\frac{2}{3}$ pounds.

23. What will be the cost of 4 hogsheads of molasses, at 1s. 2d. per quart, Mississippi currency? *Ans.* $196.

24. A farmer exchanged 28 bushels of barley, worth 5s. 8d. a bushel, with his neighbor, for corn worth 7s. a bushel; how many bushels of corn was the barley worth?

Ans. $22\frac{2}{3}$ bushels.

25. What will a load of wheat, measuring 45 bushels, be worth at 11s. a bushel, Kentucky currency?

Ans. $82.50.

26. What will 12 yards of Irish linen cost, at 4s. 9d. a yard, Pennsylvania currency? *Ans.* $7.60.

27. Bought the following bill of goods of Tradewell & Co.; how much did the whole amount to, New York currency?

4 yards of cloth	-	at	5s. 6d. per yard,
9 " calico,	-	"	1s. 4d. "
10 " ribbon,	-	"	2s. 3d. "
6 gallons molasses,	-	"	4s. 8d. per gallon,
$3\frac{1}{2}$ pounds of tea,	-	"	6s. per pound.

Ans. $13.1875.

PERCENTAGE.

179. **Per cent.** is a term derived from the Latin words *per centum*, and signifies *by the hundred*, or *hundredths*, that is, a certain number of parts of each *one hundred* parts, of whatever denomination. Thus, by 5 per cent. is meant 5 cents of every 100 cents, $5 of every $100, 5 bushels of every 100 bushels, &c. Therefore, 5 per cent. equals 5 hundredths $=.05=\frac{5}{100}=\frac{1}{20}$. 8 per cent. equals 8 hundredths $=.08=\frac{8}{100}=\frac{2}{25}$.

180. **Percentage** is such a part of a number as is indicated by the per cent.

181. The **Base** of percentage is the number on which the percentage is computed.

182. Since per cent. is any number of hundredths, it is usually expressed in the form of a *decimal* or *a common fraction*, as in the following

TABLE.

	Decimals.	Common Fractions.	Lowest Terms.
1 per cent.	— .01	— $\frac{1}{100}$	— $\frac{1}{100}$
2 per cent.	" .02	" $\frac{2}{100}$	" $\frac{1}{50}$
4 per cent.	" .04	" $\frac{4}{100}$	" $\frac{1}{25}$
5 per cent.	" .05	" $\frac{5}{100}$	" $\frac{1}{20}$
6 per cent.	" .06	" $\frac{6}{100}$	" $\frac{3}{50}$
7 per cent.	" 07	" $\frac{7}{100}$	" $\frac{7}{100}$
8 per cent.	" .08	" $\frac{8}{100}$	" $\frac{2}{25}$
10 per cent.	" .10	" $\frac{10}{100}$	" $\frac{1}{10}$
16 per cent.	" .16	" $\frac{16}{100}$	" $\frac{4}{25}$
20 per cent.	" .20	" $\frac{20}{100}$	" $\frac{1}{5}$
25 per cent.	" .25	" $\frac{25}{100}$	" $\frac{1}{4}$
50 per cent.	" .50	" $\frac{50}{100}$	" $\frac{1}{2}$
100 per cent.	" 1.00	" $\frac{100}{100}$	" 1

183. To find the percentage of any number.

1. A man having \$120, paid out 5 per cent. of it for groceries; how much did he pay out?

OPERATION.

\$120
.05
\$6.00

ANALYSIS.—Since 5 per cent. is $\frac{5}{100}$ = .05, he paid out .05 of \$120, or \$120 × 05 = \$6. Hence the

RULE. *Multiply the given number or quantity by the rate per cent. expressed decimally, and point off as in decimals.*

EXAMPLES FOR PRACTICE.

2. What is 4 per cent. of \$300? *Ans.* \$12.
3. What is 3 per cent. of \$175? *Ans.* \$5.25.
4. What is 5 per cent. of 450 pounds?
5. What is 6 per cent. of 65 gallons? *Ans.* 3.9 gal.
6. What is 9 per cent. of 200 sheep? *Ans.* 18 sheep.
7. What is 7 per cent. of \$97? *Ans.* \$6.79.
8. What is 10 per cent. of \$12.50? *Ans.* \$1.25.
9. What is 40 per cent. of 840 men? *Ans.* 336 men.
10. What is 25 per cent. of 740 miles?
11. A man having \$4000, invests 25 per cent. of it in land; what sum does he invest? *Ans.* \$1000.
12. A man bought 1500 barrels of apples, and found on opening them that 12 per cent. of them were spoiled; how many barrels did he lose? *Ans.* 180 barrels.
13. A farmer having 180 sheep, sold 45 per cent. of them and kept the remainder; how many did he sell and how many did he keep? *Ans.* He kept 99.
14. Having deposited \$1275 in bank, I draw out 8 per cent. of it; how much remains? *Ans.* \$1173.

COMMISSION.

184. An **Agent, Factor,** or **Broker,** is a person who transacts business for another.

185. A **Commission Merchant** is an agent who buys and sells goods for another.

186. **Commission** is the fee or compensation of an agent, factor, or commission merchant.

187. To find the commission or brokerage on any sum of money.

1. A commission merchant sells butter and cheese to the amount of $1540; what is his commission at 5 per cent.?

OPERATION. $1540 \times .05 = \$77$, *Ans.*

ANALYSIS. Since the commission on $1 is 5 cents or .05 of a dollar, on $1540 it is $1540 \times .05 = \$77$. Hence the

RULE. *Multiply the given sum by the rate per cent. expressed decimally; the result will be the commission or brokerage.*

EXAMPLES FOR PRACTICE.

2. What commission must be paid for collecting $3840, at 3 per cent.? *Ans.* $115.20.

3. A commission merchant sells goods to the amount of $5487.50; what is his commission at 2 per cent.? *Ans.* $109.75.

4. An agent buys 5460 bushels of wheat at $1.50 a bushel; how much is his commission for buying, at 4 per cent.? *Ans.* $327.60.

5. A commission merchant sells 400 barrels of potatoes at $2.25 a barrel, and 345 barrels of apples at $3.20 a barrel; how much is his commission for selling, at 5 per cent.?

6. An agent sold my house and lot for $6525; what was his commission at 2 per cent.?

PROFIT AND LOSS.

188. **Profit** and **Loss** are commercial terms, used to express the gain or loss in business transactions, which is usually reckoned at a certain per cent. on the prime or first cost of articles.

189. To find the amount of profit or loss, when the cost and the gain or loss per cent. are given.

1. A man bought a horse for $135, and afterward sold him for 20 per cent. more than he gave; how much did he gain?

OPERATION.

$135 × .20 = $27, *Ans.*

ANALYSIS.—Since $1 gains 20 cents, or 20 per cent., $135 will gain $135 × .20 = $27. Hence the

RULE. *Multiply the cost by the rate per cent. expressed decimally.*

EXAMPLES FOR PRACTICE.

2. Bought a horse for $150, and sold him at 15 per cent. profit; how much was my gain? *Ans.* $22.50.

3. Bought 25 cords of wood at $3.50 a cord, and sold it so as to gain 33 per cent.; how much did I make? *Ans.* $28.87½.

4. Paid 7 cents a pound for 2480 pounds of pork, and afterward lost 10 per cent. on the cost, in selling it; how much was my whole loss? *Ans.* $17.36.

5. Bought 1000 bushels of wheat at $1.25 a bushel, and sold the flour at 18 per cent. advance on the cost of the wheat; how much was my whole gain? *Ans.* $225.

6. A grocer bought 6 barrels of sugar, each containing 220 pounds, at 7½ cents a pound, and sold it at 20 per cent. profit; how much was the whole gain? *Ans.* $19.80.

SIMPLE INTEREST.

190. **Interest** is a sum paid for the use of money.

191. **Principal** is the sum for the use of which interest is paid.

192. **Rate per cent. per annum** is the sum per cent. paid for the use of $100 annually.

NOTE. The rate per cent. is commonly expressed decimally, as hundredths. (182.)

193. **Amount** is the sum of the principal and interest.

194. **Simple Interest** is the sum paid for the use of the principal only, during the whole time of the loan or credit.

195. **Legal Interest** is the rate per cent. established by law. It varies in different States, as follows:

Alabama,	8	per cent.	Minnesota,	7	per cent.
Arkansas,	6	" "	Mississippi,	3	" "
California,	10	" "	Missouri,	6	" "
Connecticut,	6	" "	New Hampshire,	6	" "
Delaware,	6	" "	New Jersey,	6	" "
Dist. of Columbia,	6	" "	New York,	7	" "
Florida,	8	" "	North Carolina,	6	" "
Georgia,	7	" "	Ohio,	6	" "
Illinois,	6	" "	Pennsylvania,	6	" "
Indiana,	6	" "	Rhode Island	6	" "
Iowa,	7	" "	South Carolina,	7	" "
Kentucky,	6	" "	Tennessee,	6	" "
Louisiana	5	" "	Texas,	8	" "
Maine	6	" "	U. S. (debts),	6	" "
Maryland,	6	" "	Vermont,	6	" "
Massachusetts,	6	" "	Virginia,	6	" "
Michigan,	7	" "	Wisconsin,	7	" "

NOTES. 1. The legal rate in Canada, Nova Scotia, and Ireland is 6 per cent., and in England and France 5 per cent.

2. When the rate per cent. is not specified in accounts, notes, mortgages, contracts, &c., the legal rate is always understood.

CASE I.

196. To find the interest on any sum, at any rate per cent., for years and months.

1. What is the interest on \$140 for 3 years 3 months, at 7 per cent.?

OPERATION.

\$140
.07
\$9.80 Int. for 1 year.
$3\frac{1}{4}$
245
2940
Ans. \$31.85 Int. for 3 yr. 3 mo.

ANALYSIS.—The interest on \$140, for 1 yr., at 7 per cent., is .07 of the principal, or \$9.80, and the interest for 3 yr. 3 mo. is $3\frac{6}{12}=3\frac{1}{4}$ times the interest for one yr., or \$9.80 $\times 3\frac{1}{4}$, which is \$31.85 the *Ans.* Hence, the following

RULE. I. *Multiply the principal by the rate per cent., and the product will be the interest for* 1 *year.*

II. *Multiply this product by the time in years and fractions of a year, and the result will be the required interest.*

EXAMPLES FOR PRACTICE.

2. What is the interest on \$48.50 for 2 years 6 months, at 6 per cent.? *Ans.* \$7.275.

3. What is the interest on \$325.41 for 3 years 4 months, at 5 per cent.? *Ans.* \$54.235.

4. What is the interest on \$279.60 for 1 year 9 months, at 7 per cent.? *Ans.* \$34.251.

5. What is the amount of \$26.84 for 2 yr. 6 mo., at 5 per cent.? *Ans.* \$30.195.

6. What is the amount of \$200 for 1 yr. 9 mo., at 7 per cent? *Ans.* \$224.50.

7. What is the interest on \$750 for 1 year 3 months, at 5 per cent.? *Ans.* \$46.875.

CASE II.

197. To find the interest on any sum, for any time, at any rate per cent.

Obvious Relations between Time aud Interest.

I. The interest on any sum for 1 year, at 1 per cent., is .01 of that sum, and is equal to the principal with the seperatrix removed two places to the left.

II. A month being $\frac{1}{12}$ of a year, $\frac{1}{12}$ of the interest on any sum for 1 year is the interest for 1 month.

III. The interest on any sum for 3 days is $\frac{3}{30}=\frac{1}{10}=.1$ of the interest for 1 month, and any number of days may readily be reduced to *tenths* of a month by dividing by 3.

IV. The interest on any sum for 1 month, multiplied by any given time expressed in months and tenths of a month, will produce the required interest.

1. What is the interest on \$306 for 1 yr. 6 mo. 12 da., at 7 per cent?

OPERATION.

1 yr. 6 mo. 12 da.=18.4 mo.

12)\$3.060

\$.255
18.4

1020
2040
255

4.6920
7

\$32.8440 *Ans.*

ANALYSIS. Removing the seperatrix in the given principal two places to the left, we have \$3.06, the interest on the given sum for 1 year at 1 per cent. (I). Dividing this by 12, we have \$.255, the interest for 1 month, at 1 per cent. (II). Multiplying this quotient by 18.4, the time expressed in months and decimals of a month. (III). we have \$4.692, the interest on the given sum for the given time, at 1 per cent. (IV). And multiplying this product by 7, the rate per cent. we have \$32.844, the required interest. Hence,

RULE. I. *Remove the separatrix in the given principal two places to the left; the result will be the interest for* 1 *year at* 1 *per cent.*

II. *Divide this interest by* 12; *the result will be the interest for* 1 *month, at* 1 *per cent.*

III. *Multiply this interest by the given time expressed in months and tenths of a month; the result will be the interest for the given time, at* 1 *per cent.*

IV. *Multiply this interest by the given rate; the product will be the interest required.*

EXAMPLES FOR PRACTICE.

2. What is the interest on $34.25 for 3 yr. 8 mo. 15 da., at 5 per cent.? *Ans.* $6.35.

3. What is the interest on $260 for 9 mo. 3 da., at 6 per cent.? *Ans.* $11.826.

4. What is the interest on $450, at 6 per cent. for 10 mo. 18 days? *Ans.* $23.85.

5. What is the interest on $372 for 1 yr. 10 mo. 15 days, at 7 per cent.? *Ans.* $48.825.

6. What is the interest on $221.75 for 3 yr. 7 mo. 6 da., at 7 per cent.? *Ans.* $55.88.

8. What is the interest on $267.27 for 6 mo. 24 days, at 6 per cent.? *Ans.* $9.086.

9. What is the interest on $365 for 2 mo. 3 days, at 6 per cent.? *Ans* .$3.83.

10. What is the interest on $785.10, for 1 yr. 6 months 18 days, at 5 per cent.? *Ans.* $60.845.

11. On $450 for 3 yr. 7 months, at 8 per cent.?

12. What is the interest on $600 for 2 yr. 8 mo., at 7 per cent.? *Ans.* $112.

13. What is the amount of $1000 for 9 mo. 15 days, at 7 per cent.? *Ans.* $1055.414.

14. What is the interest on $860 for 6 mo. 6 days, at 6 per cent.? *Ans.* $26.66.

15. What is the interest on $137.45 for 8 mo. 27 days, at 6 per cent.?

16. Find the amount of $875 for 1 yr. 6 mo. at 3 per cent.? *Ans.* $914.375.

17. Find the amount of $350 for 9 mo., at 4 per cent? *Ans.* $360.497.

18. Find the amount of $8.50 for 1 yr. 9 mo. 12 da., at 6 per cent.? *Ans.* $9.409.

19. Find the amount of $457 for 1 yr. 4 mo. 24 da., at 6 per cent.? *Ans.* $495.388.

20. Find the amount of $650 for 3 yr. 10 mo. 21 days at 7 per cent.? *Ans.* $827.009.

21. What is the interest on $79 for 15 mo., at 7 per cent.? *Ans.* $6.912×.

22. Find the amount of $.86 for 5 mo., 7 per cent. *Ans.* $.885.

23. What is the interest on $78.75 for 1 yr. 9 mo., at 4 per cent.? *Ans.* $5.5125.

24. What is the interest on $1750 for 30 days, at 9 per cent.? *Ans.* $13.125.

25. What is the interest on $3654.25 for 33 days, at 10 per cent.? *Ans.* $33.497.

26. Find the amount of $269.50 for 120 days, at 7 per cent.? *Ans.* $275.788.

27. Find the amount of $1625 for 1 yr. 6 mo., at 8 per cent.? *Ans.* $1820.

NOTE.—For a full treatise of Percentage in all its applications to the business transactions of life, and also for the developement and application of those subjects ordinarily treated by arithmetic, the pupil is referred to the Author's Progressive Practical, and Progressive Higher Arithmetics.

PROMISCUOUS EXAMPLES.

1. Multiply the difference between 876042 and 834260 by 176. *Ans.* 7353632.

2. To 47320 add three times the difference between 46270 and 31032. *Ans.* 93034.

3. From 212462+432046, take 517240−230124.

4. Divide the sum of 4802+56010+20342 by 4 times the difference between 1200 and 1082.

Ans. $171\frac{221}{236}$.

5. What is the difference between 1824624+15624 and 896042−12342? *Ans.* 956548.

6. What is the difference between 3426×284 and 200104? *Ans* 772880.

7. What is the difference between 3931476÷556 and 14×875? *Ans.* 5179.

8. How many times can 36 be subtracted from 11772? *Ans.* 327.

9. How many times can 8×27 be taken from 1554768?

10. Divide 420×216 by 43756−42851.

Ans. $100\frac{44}{181}$.

11. Multiply 3 times the sum of 4624+1036 by times the difference of 375−296. *Ans.* 2682840.

12. What is the difference between 5 times 2.5, and 5×2.5? *Ans.* 11.25.

13. Multiply 4.05+.025+1.8 by 2−1.875.

14. Divide 5 by .8×.025. *Ans.* 250.

15. How many times can 1.05 be taken from 4.725?

Ans. 4.5 times.

16. To .02 times 32.5 add 5.7 times 16.04−12.0026.

Ans. 23.66318.

17. What is the difference between .675÷.15 and .23 × .009? *Ans.* 4.49793.

18. A farmer sold a horse for $140, a cow for $25, and 28 sheep at $2,50 a head; how much more did he receive for the horse than for the cow and sheep? *Ans.* $45.

19. A young lady having $75, went out shopping, and bought 14 yards of silk for a dress, at $1,50 a yard, a shawl for $15,75, a bonnet for $8, a pair of gloves for $1.125, and a pair of shoes for $1,75; how much money had she remaining? *Ans.* $27.37½.

20. A grocer bought 12 firkins of butter, each containing 56 pounds, at 14 cents a pound; he afterward sold 5 firkins, at 16 cents, and 7 firkins, at 18 cents a pound; how much was his whole gain? *Ans.* 21.28.

21. A miller sold 256 barrels of flour, at $6.80 a barrel, which was $475.60 more than the wheat from which it was made, cost him; what was the cost of the wheat? *Ans.* $1265.20.

22. An estate worth $25640, has demands against it to the amount of $9376; after these claims are paid, the remainder is to be divided equally among 5 individuals; how much will each receive? *Ans.* $3252.80

23. If 15 tons of hay cost $311.70, how much will 1 ton cost? *Ans.* $20.78.

24. Paid $1.24 for 15.5 pounds of beef; how much was the price per pound? *Ans.* $.08.

25. A farmer exchanged 21 bushels of wheat, at $2 a bushel, for cloth worth $3 a yard; how many yards did he receive? *Ans.* 14 yards.

26. A man having labored for a farmer 1 year, at $15 a month expended the year's wages for cows, at $18 each; how many cows did he buy? *Ans.* 10

27. What will be the cost of 3 hogsheads of sugar, each weighing 15 cwt., at 8 cents a pound? *Ans.* $360.

28. How many bushels of wheat, at $1.12 a bushel, can be bought for $81.76? *Ans.* 73.

29. If 140 barrels of apples cost $329, how much is the cost per barrel? *Ans.* $2.35.

30. At $.825 per bushel, how many bushels of corn can be bought for $264? *Ans.* 320.

31. If 25 yards of cloth can be bought for $125.25, how many yards can be bought for $751.50? *Ans.* 150.

32. If 150 bushels of wheat cost $435, how much will 311 bushels cost? *Ans.* $901.90.

33. If 250 pounds of tea cost $135, what is the price per pound? *Ans.* $.54.

34. If 13 spoons be made from 2 lb. 10 oz. 9 pwt. of silver, what will be the weight of each?

Ans. 2 oz. 13 pwt.

35. If a man travels 20 mi. 3 fur. 36 rd. in a day, how far will he travel in 61 days at the same rate?

Ans. 1249 mi. 5 fur. 36 rd.

36. If I put 376 gal. 3 qt. 1 pt. of cider into 9 equal casks, how much do I put into each cask?

37. If a family use $1\frac{1}{8}$ pounds of tea in 1 month, how much would they use in 1 year? *Ans.* $13\frac{1}{2}$ pounds.

38. What would be the cost of 565 pounds of butter at $12\frac{1}{2}$ cents a pound? *Ans.* $70.625.

39. At $4.25 per bushel how much clover-seed can be bought for $11.6875? *Ans.* 2.75 bushels.

40. At $\frac{1}{16}$ of a dollar a pound, what will be the cost of 12 pounds of sugar? *Ans.* $.75.

41. At $\frac{3}{8}$ of a dollar a yard, what will be the cost of $40\frac{1}{4}$ yards of cloth? *Ans.* $15.30.

42. How many cubic yards of earth must be thrown from a cellar 40 ft. long, 30 ft. wide, 6 ft. deep; and what will be the cost of the excavation, at $12\frac{1}{2}$ cents a cubic yard? *Ans.* $266\frac{2}{3}$ cubic yards; $\$33.33\frac{1}{3}$.

43. If 6 pounds of cheese cost $\$\frac{4}{5}$, how much will 10 pounds cost? *Ans.* $\$1\frac{1}{3}$.

44. How much wheat at \$1.25 a bushel, must be given for 50 bushels of corn at \$.70 a bushel?

45. At 10 cents a pint, how much will 189 gallons of molasses cost? *Ans.* \$151.20.

46. At 15 cents a pound, how much will $\frac{4}{25}$ of a pound of coffee cost? *Ans.* $2\frac{2}{5}$ cents.

47. If 3 gallons of molasses cost $\$\frac{5}{6}$, how many gallons can be bought for \$4? *Ans.* $14\frac{2}{5}$.

48. At $\$7\frac{1}{2}$ a firkin, how many firkins of butter can be bought for \$33? *Ans.* $4\frac{2}{5}$.

49. If $\frac{1}{7}$ of a yard of cloth cost $\$\frac{4}{7}$, what will one yard cost? *Ans.* $\$2\frac{4}{7}$.

50. At \$3 a barrel, how many barrels of cider can be bought for $\$8\frac{3}{5}$? *Ans.* $2\frac{13}{15}$ barrels.

51. What part of 100 pounds is 16 pounds?

Ans. $\frac{4}{25}$.

52. How much wood in a load 10 ft. long, $3\frac{1}{2}$ ft. wide, and 4 ft. high? *Ans.* 1 Cd. 12 cu. ft.

53. How many tons of coal may be bought for \$346.125 at \$9.75 per ton? *Ans.* 35.5 tons.

54. What is the interest on \$136.80 for 1 yr. 11 mo., at 7 per cent.? *Ans.* \$18.354.

55. What will be the cost of .6 of a gallon of wine, at \$.65 a gallon? *Ans.* \$.39.

56. A owns $\frac{5}{7}$ of a flouring mill, and sells $\frac{2}{5}$ of his share to B; what part of the whole has he left?

57. If 2 yards of cloth cost $\$6\frac{3}{4}$, how much will 9 yards cost? *Ans.* $\$30\frac{3}{8}$.

58. What will $\frac{1}{2}$ of $\frac{5}{8}$ of a barrel of flour cost at $\$7\frac{1}{5}$ per barrel? *Ans.* $\$2\frac{1}{4}$.

59. If 1 acre of land yield 1 T. 9 cwt. 1 qr. 22 lb. of hay, how much will 18 acres yield?

60. A speculator bought 1575 barrels of potatoes, and upon opening them, he found 15 per cent. of them spoiled; how many barrels did he lose? *Ans.* 236.25.

61. How many steps of 30 inches each, must a person take in walking 10 miles? *Ans.* 21120.

62. A man bought 12 bushels of chestnuts, at $\$4.50$ a bushel, and sold them at 12 cents a pint; how much was his whole gain? *Ans.* $\$8.16$.

63. What is the interest on $\$300$, for 10 mo. 21 days, at 6 per cent.? *Ans.* $\$16.05$.

64. An agent in Chicago, purchased 5450 bushels of wheat, at $\$.82$ a bushel; what was his commission at 2 per cent. on the purchase money? *Ans.* $\$89.38$.

65. A vessel loaded with 4500 bushels of corn, was overtaken by a storm at sea, and it was found necessary to throw overboard 25 per cent. of her cargo; what was the whole loss, at 60 cents a bushel? *Ans.* $\$675$.

66. A grocer bought 2 hogsheads of molasses, at $37\frac{1}{2}$ cents a gallon, and sold it at 20 per cent. advance on the cost; how much was his whole gain? *Ans.* $\$9.45$.

67. If $\frac{5}{7}$ of acres of land is worth $\$60$, what is the value of 1 acre? *Ans.* $\$84$.

68. If $1\frac{1}{3}$ bushels of wheat sow an acre of land, how many acres will 12 bushels sow? *Ans.* 9 acres.

69. If a farm is worth $\$3840$, how much is $\frac{5}{8}$ of it worth? *Ans.* $\$2400$.

70. If 17 kegs of nails weigh 27 cwt. 3 qrs. 23 lbs. 3 oz., long ton weight, how much will 1 keg weigh?

71. If a bushel of apples cost $\frac{2}{5}$ of a dollar, how many may be bought for $\frac{3}{7}$ of a dollar?

72. Divide $\frac{1}{8}$ of $\frac{2}{3}$ by $\frac{2}{3}$ of $\frac{3}{4}$? *Ans.* $\frac{1}{6}$.

73. What is the amount of $620 for 4 yr. 3 mo., t 6 per cent.? *Ans.* $778.10.

74. What is the brokerage on $5462, at 4 per cent.,

75. How many pounds of butter at $13\frac{1}{2}$ cents a pound, must be given for 1230 pounds of sugar at 8 cents a pound? *Ans.* $728\frac{8}{9}$ pounds.

76. Divide 168 bu. 1 pk. 6 qt. of corn equally among 35 persons. *Ans.* 4 bu. 3 pk. 2 qt.

77. What will be the cost of lathing and plastering overhead, a room 36 feet long and 27 feet wide, at 28 cents a square yard? *Ans.* $30.24.

78. How much land at $2.50 an acre, must be given in exchange for 360 acres, at $3.75 an acre?

79. What is the amount of $564.58, for 3 yr. 5 mo. 12 da., at 6 per cent? *Ans.* $681.448.

80. How much sugar at 9 cents a pound, should be given for $6\frac{1}{2}$ cwt. of tobacco, at 14 cents a pound?

81. How many times may a jug which holds $\frac{7}{8}$ of a gallon, be filled from a cask containing 128 gallons?

82. A man having $25000, invested 30 per cent. of it in bonds and mortgages, 45 per cent. of it in bank stocks, and the remainder in railroad stock; how much did he invest in railroad stock?

Ans. $6250.

83. How many times can a box holding 4 bu. 3 pk 2 qt., be filled from 336 bu. 3 pk. 4 qt.?

Ans. 70.

84. How many cords of wood in 17 piles, each 11 feet long, 4 feet wide, and 6 feet high?

85. If the price of 1 acre of land is $\$32\frac{3}{4}$, what is the value of $\frac{7}{8}$ of an acre? *Ans.* $\$28\frac{21}{32}$.

86. What number of times will a wheel 14 ft. 10 in. in circumference, turn round in traveling 11 mi. 6 fur 15 rd. 12 ft. 6 in.? *Ans.* 4200.

87. A man bought a farm of 136 acres, at $94 an acre; he paid $475 for fencing and the improvements, and then sold it for 14 per cent. advance on the whole cost; how much was his whole gain?

Ans. $1856.26.

88. If 36.48 yards of cloth cost $54.72, how much will 14.25 yards cost? *Ans.* $21.375.

89. If $13.342 will pay for 17.5 bushels of barley, how many bushels can be bought for $76.24?

Ans. 100 bushels.

90. A lady having $40.50, spent 40 per cent. of it for dry goods; how much had she left?

Ans. $24.30.

91. A gentleman bought a house and lot for $6425; in the course of five years it increased in value 110 per cent. how much was the property then worth?

Ans. $13492.50.

92. How much will a broker charge to change $560 uncurrent money for current money, at 3 per cent.?

Ans. $16.80.

93. If 4 hogsheads of wine cost $181.44, what will be the cost of 1 pint? *Ans.* 9 cents.

94. What will 5 casks of rice cost, each weighing 165 pounds, at 3d per pound, Georgia currency?

Ans. $44.198.

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